Comparing representations

towards a strong generative capacity for phonology

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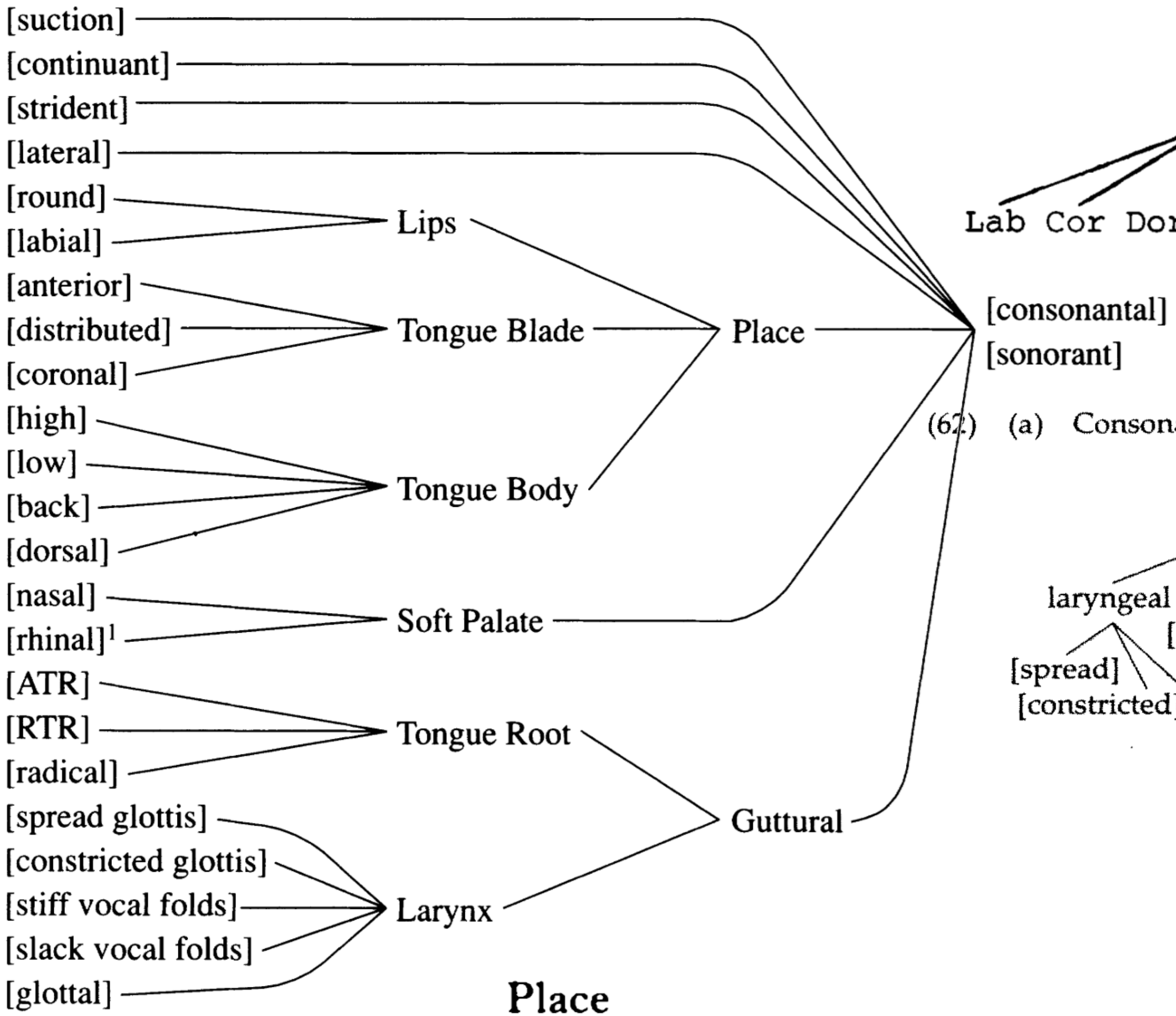
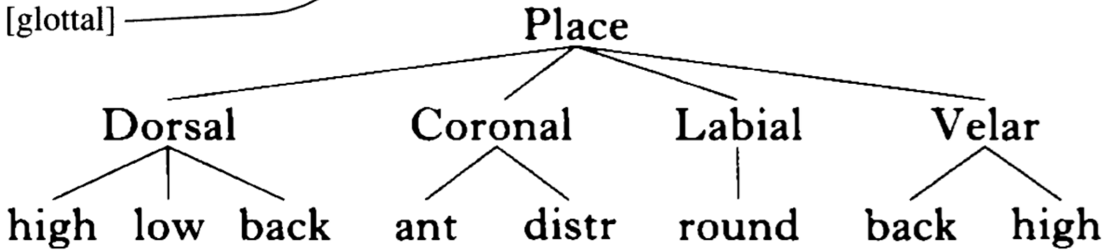
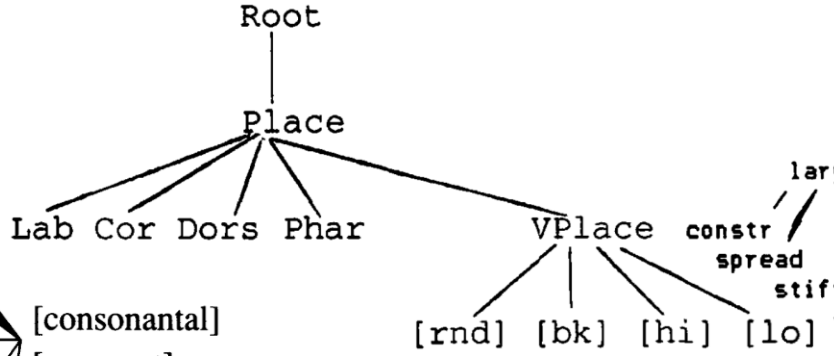
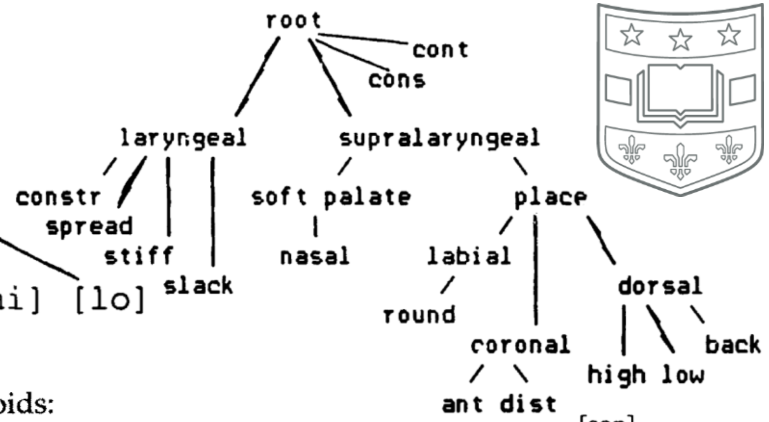
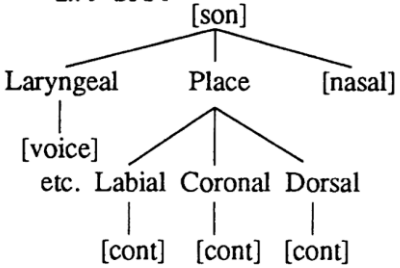


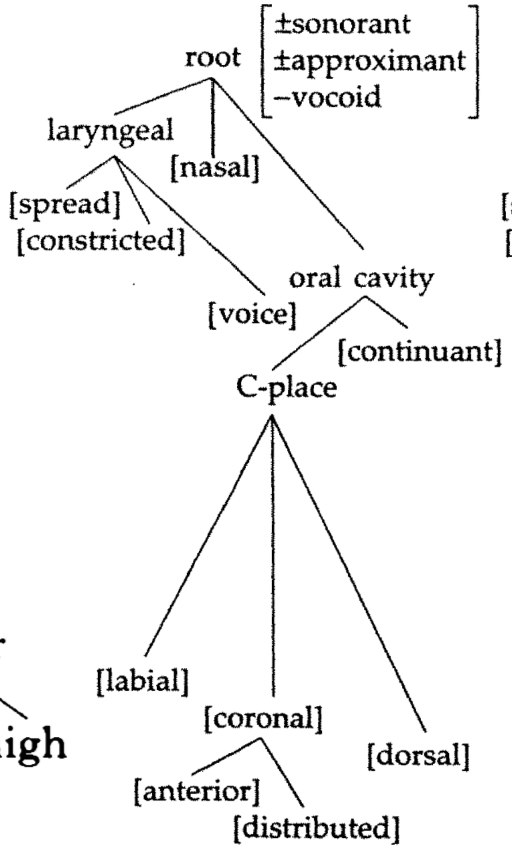




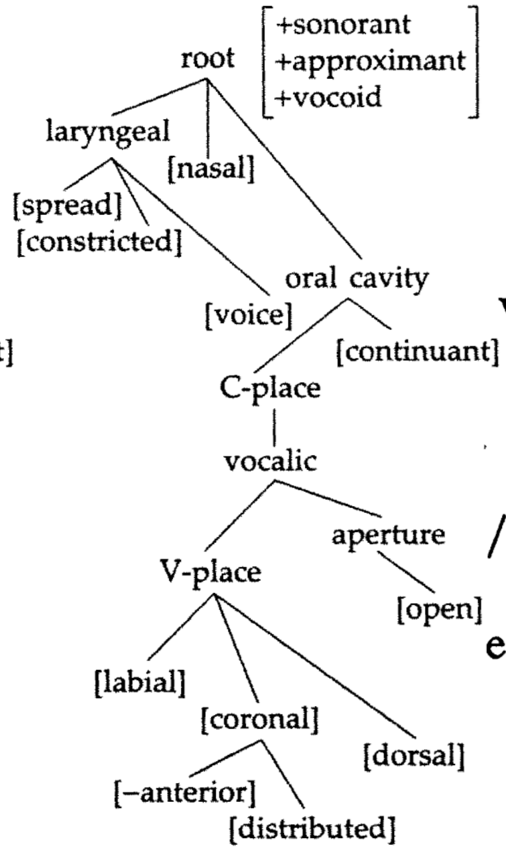


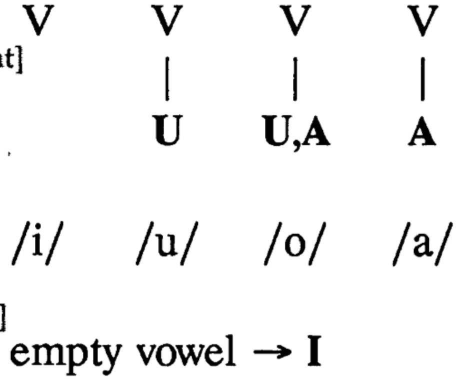


(62) (a) Consonants:

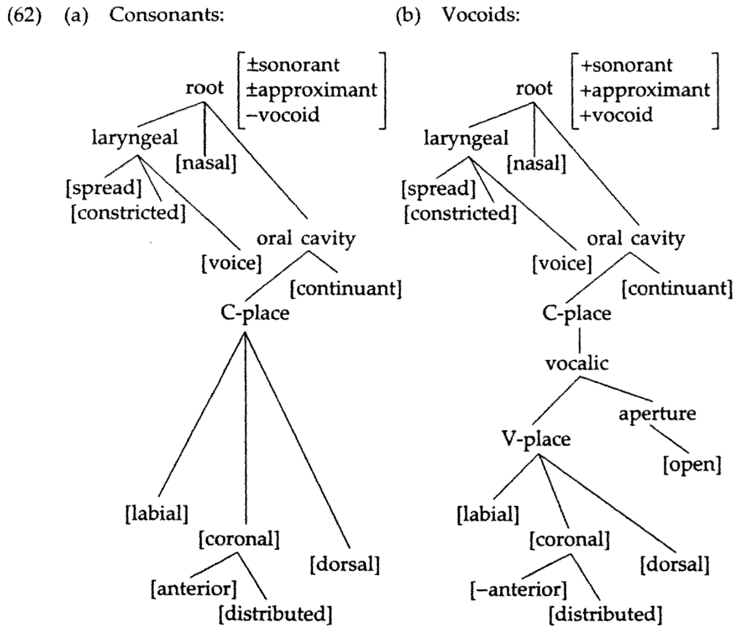


(b) Vocoids:

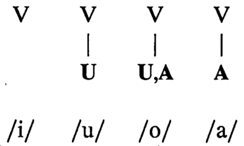




consonants and vowels probably should be natural classes

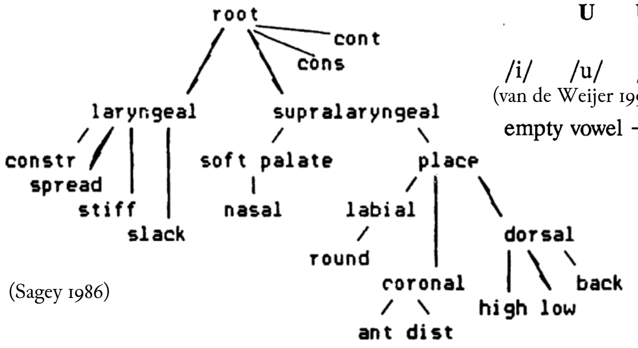


(Clements and Hume 1995)



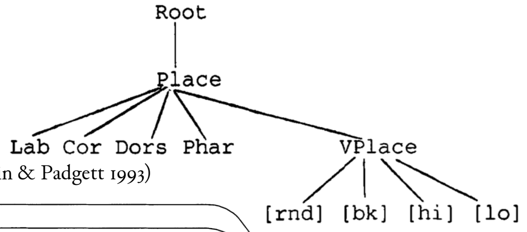
(van de Weijer 1996)

empty vowel → I

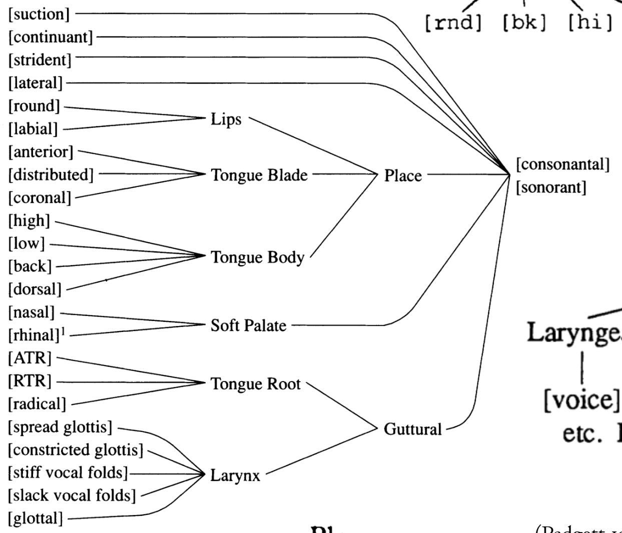


(Sagey 1986)

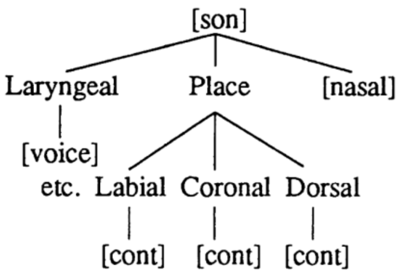
consonants and vowels probably *shouldn't* be natural classes



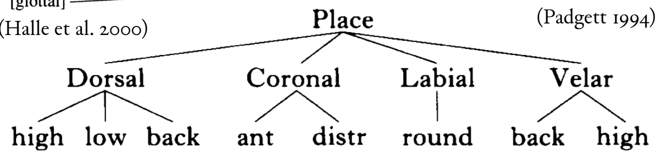
(Ní Chiosáin & Padgett 1993)



(Halle et al. 2000)



(Padgett 1994)



(Odden 1991)

big questions

- how can we formally compare phonological representations?
- what can we learn from these comparisons?
- what do we care about as linguists?
- why care about anything?

medium answers

- two theories can be shown to be **formally equivalent** using logic and model theory
  - given two representations A and B, a **transduction** between A and B means that any linguistic rule given with structure A can be translated into structure B, and vice versa
  - Strother-Garcia (2019), Danis & Jardine (2019), Oakden (2020), a.o.

Figure 4.5: $\mathcal{M}^{flat}_{plenty}$

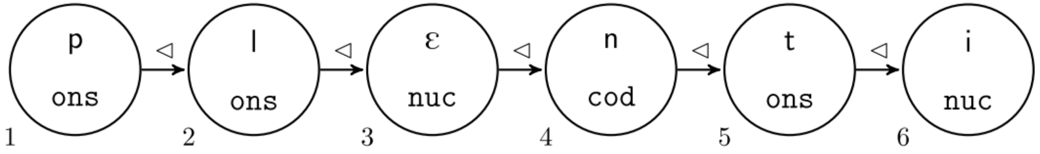
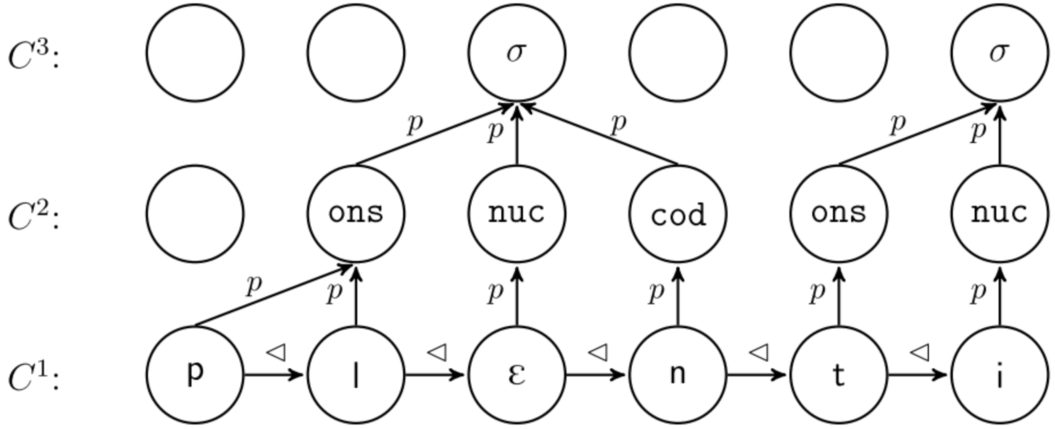


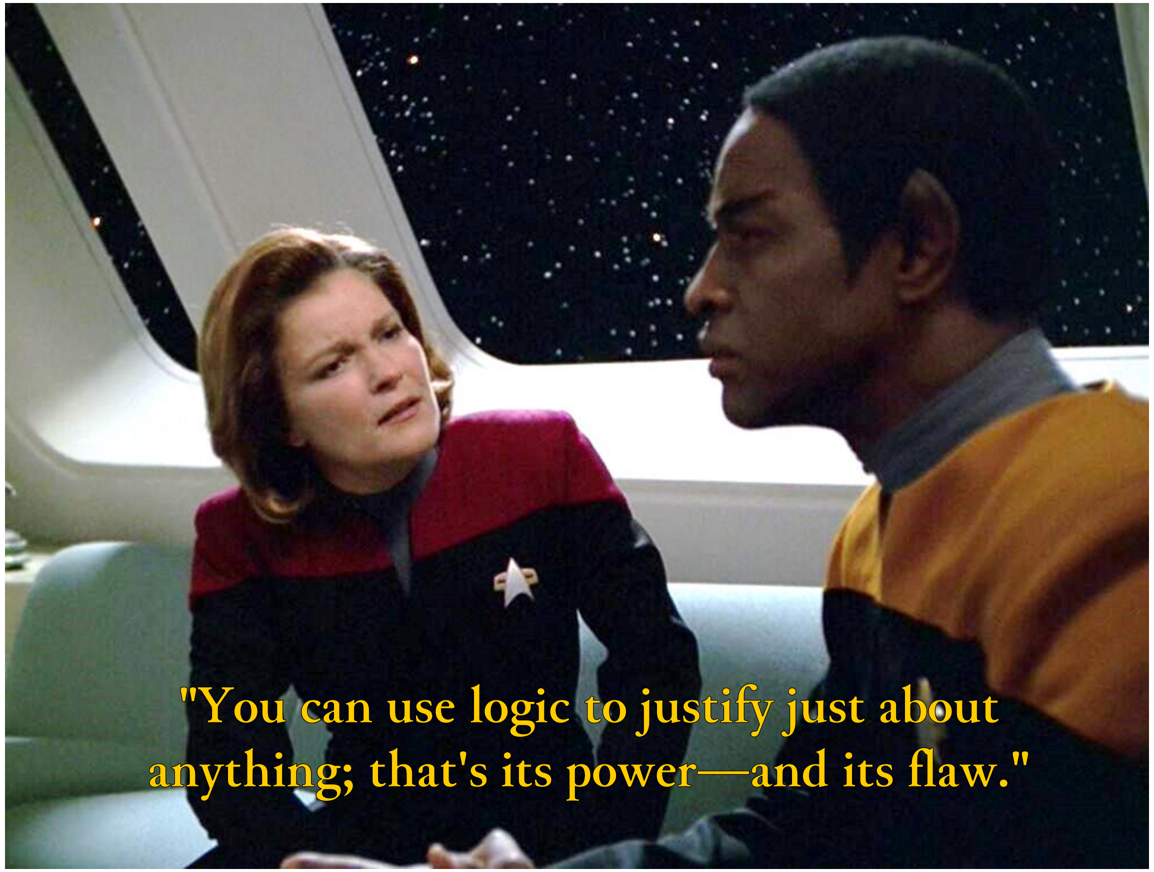
Figure 4.16: $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$ fully specified



(Strother-Garcia 2019)

medium answers

- not every transduction preserves ideas of ***linguistic*** equivalence
  – process should respect natural classes, which may be lost in certain transductions
- the property of a **natural-class preserving transduction** is defined to find those logically equivalent representations that also share linguistic intuitions



introduction

foundations

example transduction

existing transductions

broader significance

conclusion

natural classes

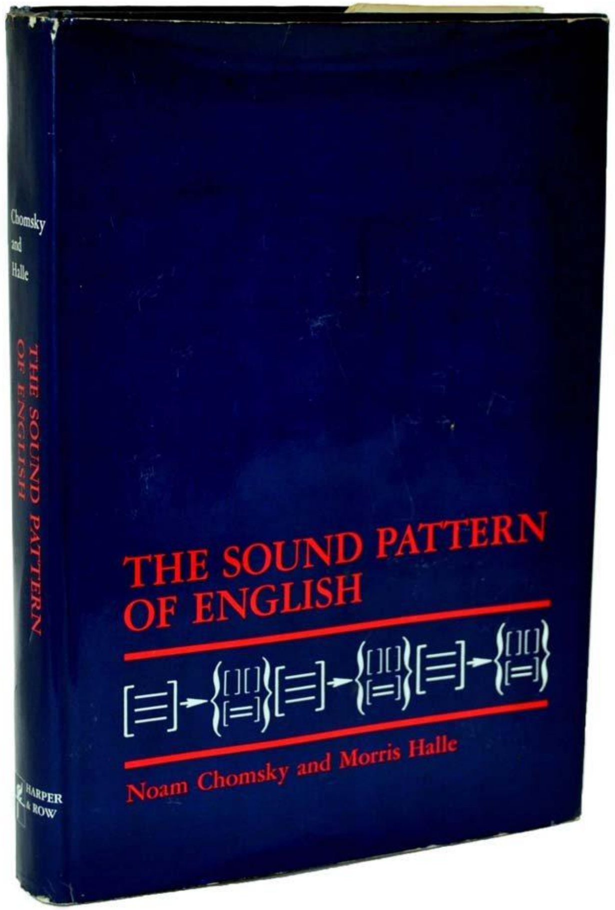
- segments have structure (e.g. features)
- some segments potentially share structure
- a **natural class** is an exhaustive set of all segments that share some piece of structure

why do we care about natural classes?

natural classes

"In view of this, if a theory of language failed to provide a mechanism for making distinctions between more or less natural classes of segments, **this failure would be sufficient reason for rejecting the theory as being incapable of attaining the level of explanatory adequacy."**

(Chomsky & Halle 1968: 355)



natural classes

"This combinability of features allows phonology to construct complex symbols from an inventory of simple parts, and provides an explanation for the so-called natural class behavior—**different structures can behave alike because they contain identical substructures**."

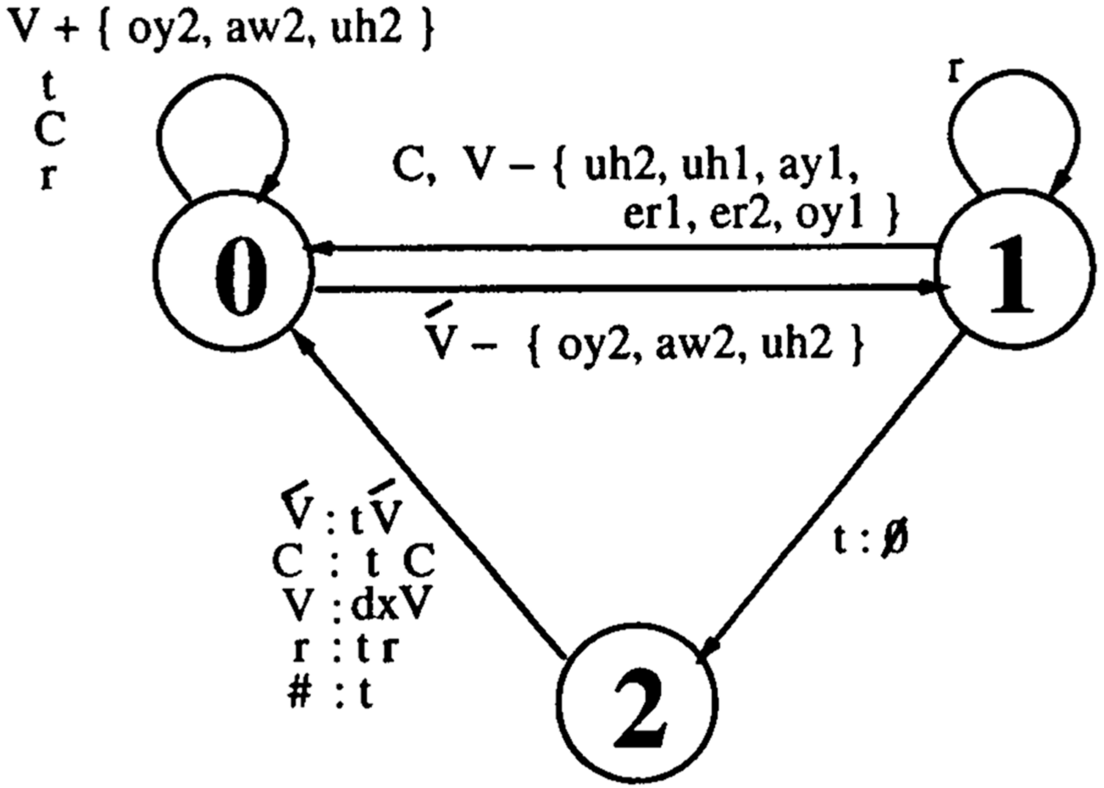
" In Logical Phonology (see section 3), rules refer to natural classes by definition: **a statement that cannot be formulated in terms of natural classes is not a rule**."

(Volenec & Reiss 2020: 22, 28)

natural classes as a computational learning bias

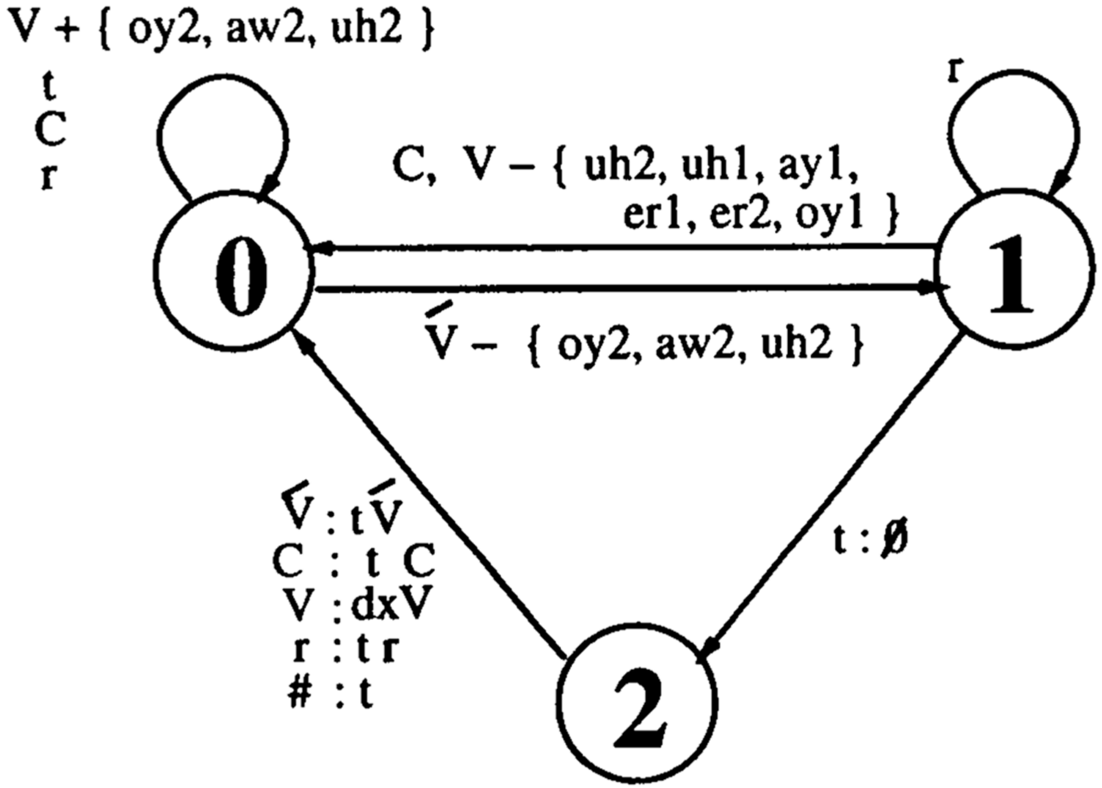
"Without an ability to **use knowledge about phonological features to generalize across phones**, OSTIA's transducers have missing transitions for certain phones from certain states. This causes errors when transducing previously unseen words after training is complete."

(Gildea & Jurafsky 1996)

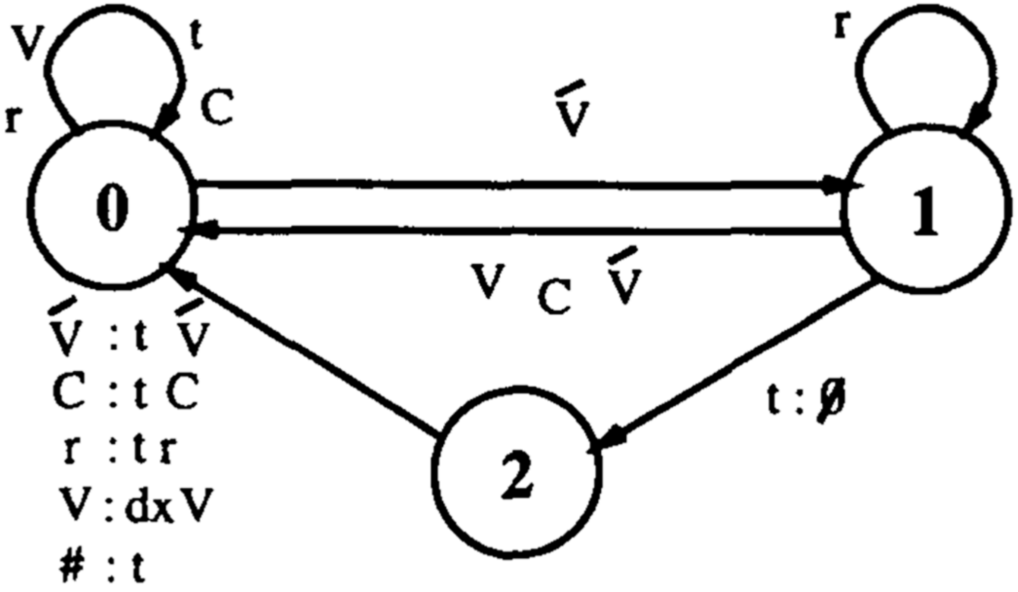


preliminary FST trained on English flapping alternations

natural classes as a computational learning bias



transducer learned with no natural class knowledge

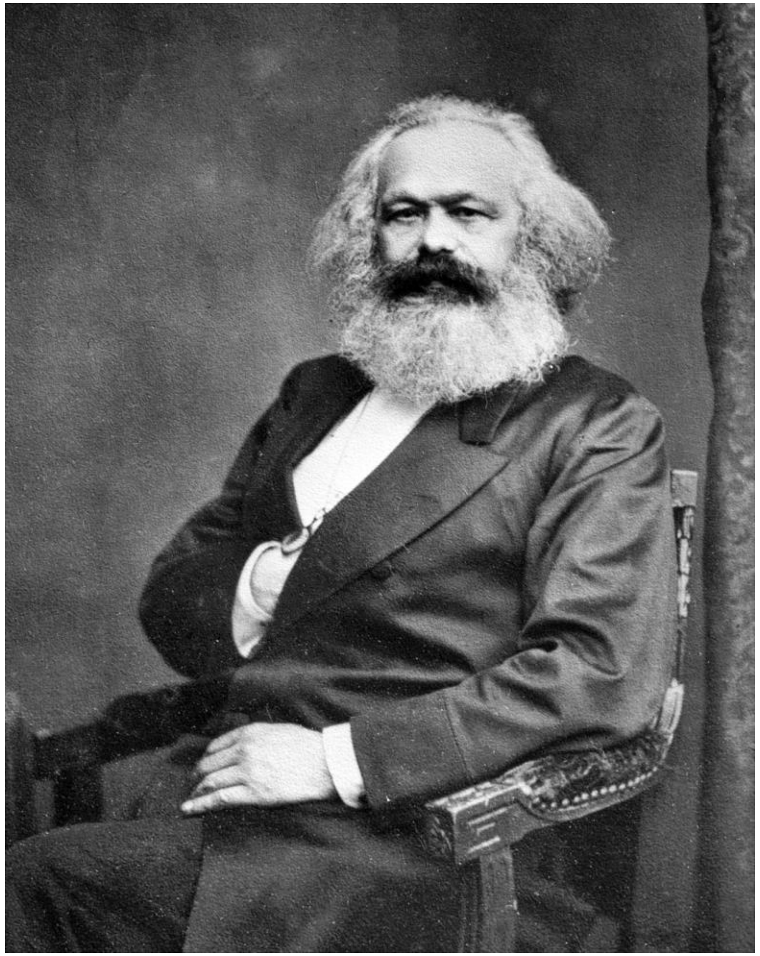


transducer learned with knowledge of natural classes

natural classes

"...that consequently the whole history of mankind [...] has been a history of class struggles."

(Marx 1848:8)



natural class-preserving transductions

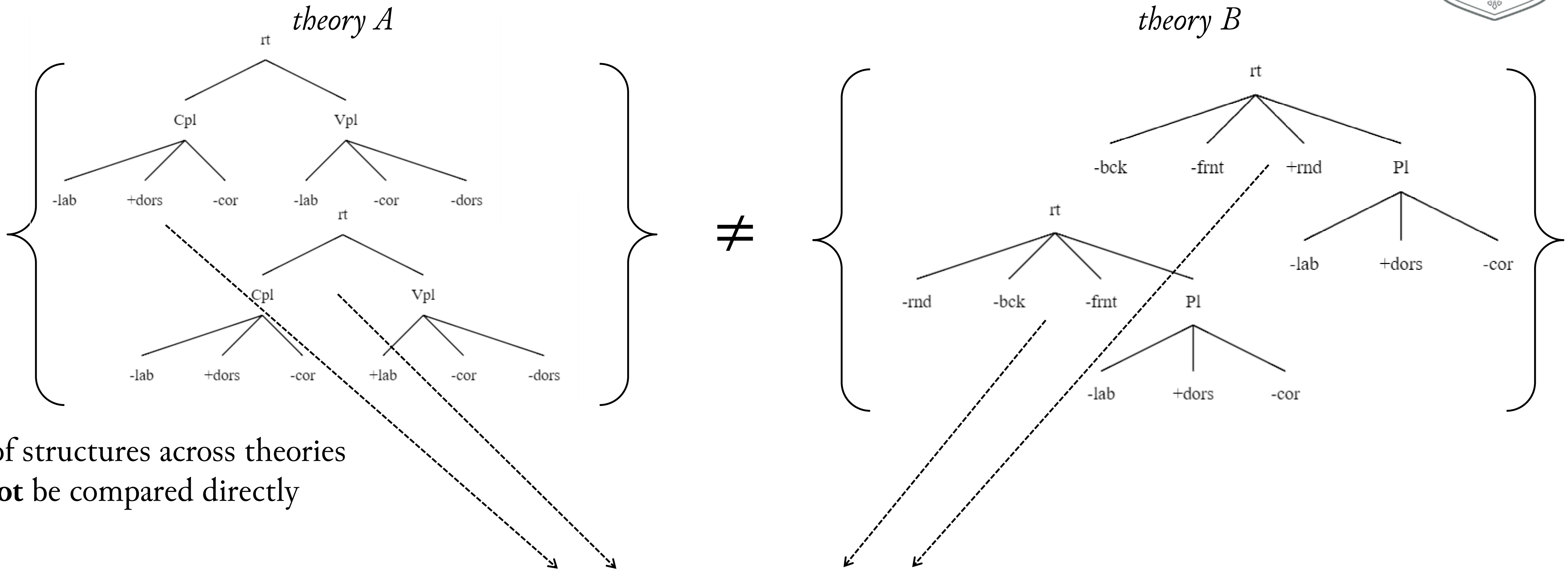
a transduction between two representational theories A and B is **natural class-preserving**🌳 iff the set of all natural class extensions of A exactly match those of B

natural class-preserving transductions

- a **natural class** is the set of all structures in some theory that have some substructure in common
  - the natural class defined by V-Place | [+labial] is the set of all structures that have a subgraph isomorphic to V-Place | [+labial] preserving the unary labeling relation

- a **natural class extension** is formed by applying a bijection from structures to atomic symbols for some natural class as defined above
  - define a mapping between each licit structure of a theory and some atomic character (e.g. IPA symbol)
  - a set of *structures* can then map to a representative set of atomic *symbols*

natural class extension



sets of structures across theories **cannot** be compared directly

$$\{\ k\ k^w\ \} = \{\ k\ k^w\ \}$$

so each structure is mapped to the appropriate **atomic symbol**

assimilation

assimilation operates over like things

Sharing is Caring 🤝

the target and trigger of an assimilation process should form a natural class *as a result of that assimilation process*

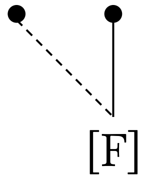
(stronger version: *all* structural changes must be natural class forming?)

assimilation: **sharing is caring**

- linear rule-based systems:
  - assimilation rules involve alpha notation across the same feature label (Chomsky and Halle 1968, Hyman 1974, a.o.)
- autosegmental/non-linear representations:
  - assimilation is spreading (Hayes 1986), Clements & Hume 1995, a.o.)
- constraint-based systems (e.g. OT):
  - multiple segments targeted by Agree constraint for a single feature (Lombardi 1999, Bakovic 2000, a.o.)

$$[G] \rightarrow [\alpha F] / \_\_ [\alpha F]$$

target trigger



/-F+F/	Agree[F]	Ident[F]
-F+F	*	
+F+F		*

assimilation: **sharing is caring** 🤝

- Clements & Hume (1995):
  - "Phonological rules perform single operations only." (p. 250)
  - "In the present model, in contrast, assimilation rules are characterized as the association (or "**spreading**") **of a feature or node F** of segment A to a neighboring segment B..." (p. 258)

- if assimilation is the result of spreading (the addition of an association relation), then it directly follows from this that the resulting segments **must** have shared structure and therefore constitute a nontrivial natural class

the general argument

1. **if** we assume a nontrivial theory of segmental structure, **and**
2. **if** we assume for assimilation that **sharing is caring** 🤝
3. **then** the range of possible assimilation processes is restricted

further:

4. **if** two theories are shown to be logically equivalent, **and**
5. **if** this transduction is not **natural class-preserving**🌳
6. **then** the two theories do not make the same empirical predictions (by 2&3)
7. **then logical equivalence is not sufficient for linguistic equivalence**

comparing theories

unified place theory

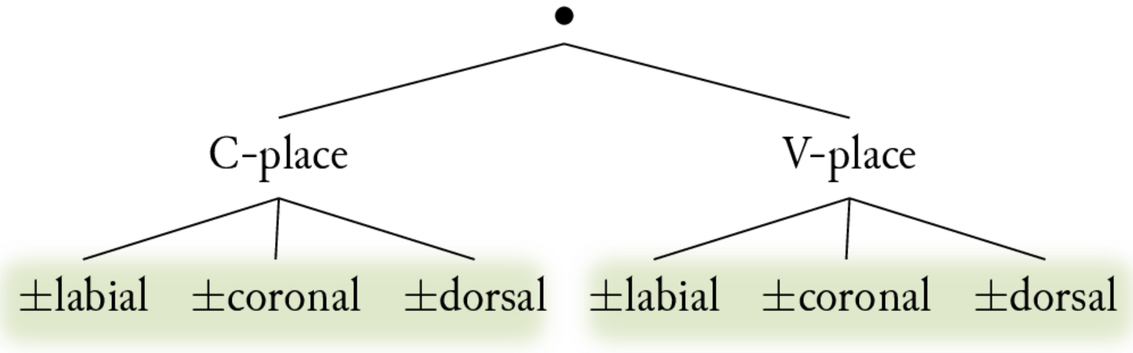
- consonants and vowels share representational primitives
  - e.g. LABIAL C-place, LABIAL V-place
- Sagey (1986), Clements & Hume (1995), a.o.

vowel features theory

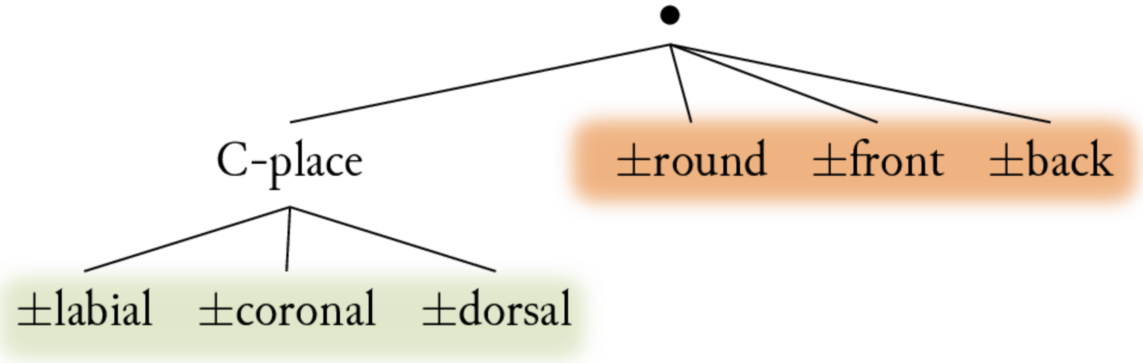
- vowel place is largely defined by primitives **not** used to describe consonant place
  - e.g. [+back], [-round]
- Odden (1991), Ni Chiosain & Padgett (1993), Halle et al. (2000), a.o.

comparing theories

unified place theory



v-features theory



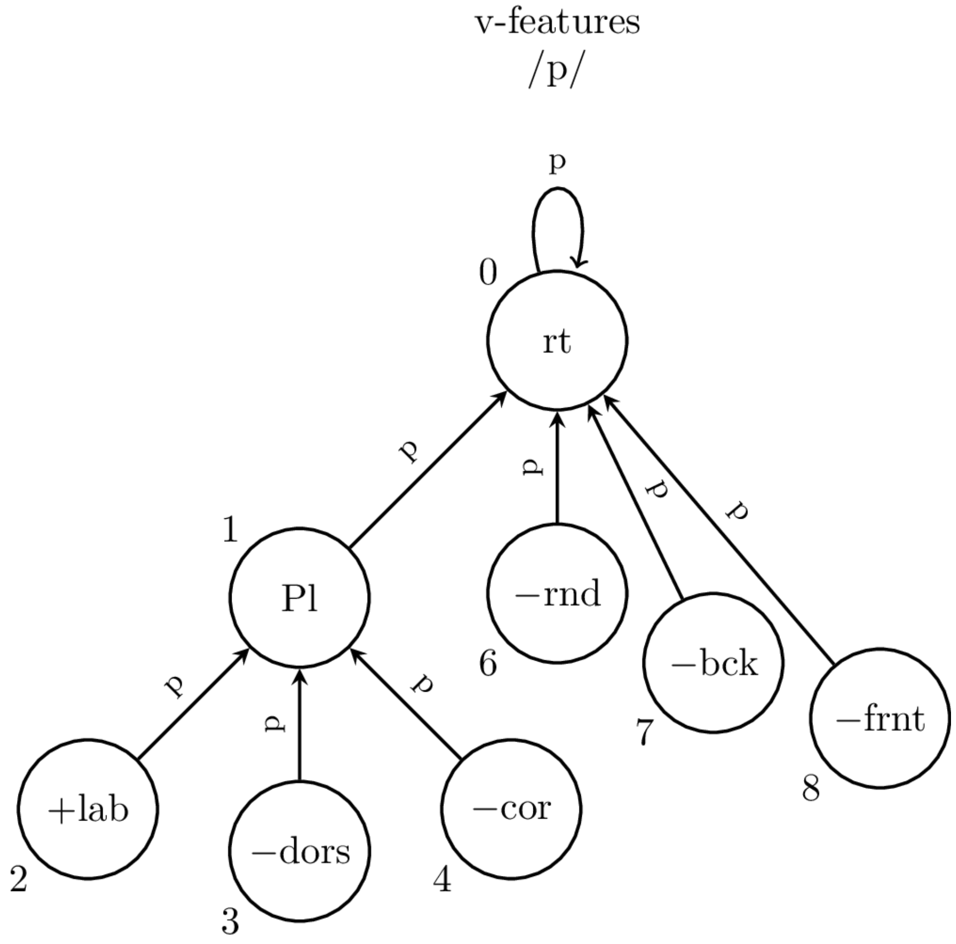
crucial difference: **unified** uses same feature labels for vocalic and consonantal contrasts

orthogonal issues:

- binary vs. privative features
- underspecification

comparing theories

- each theory is cast as a **finite model** defining the domain of nodes, relations, and functions in each
- each model defines a **logical language** for each theory of representation
- a **transduction** translates all relations & functions in one model to the other
- any sentence/rule/constraint expressible in one model is therefore expressible in the other

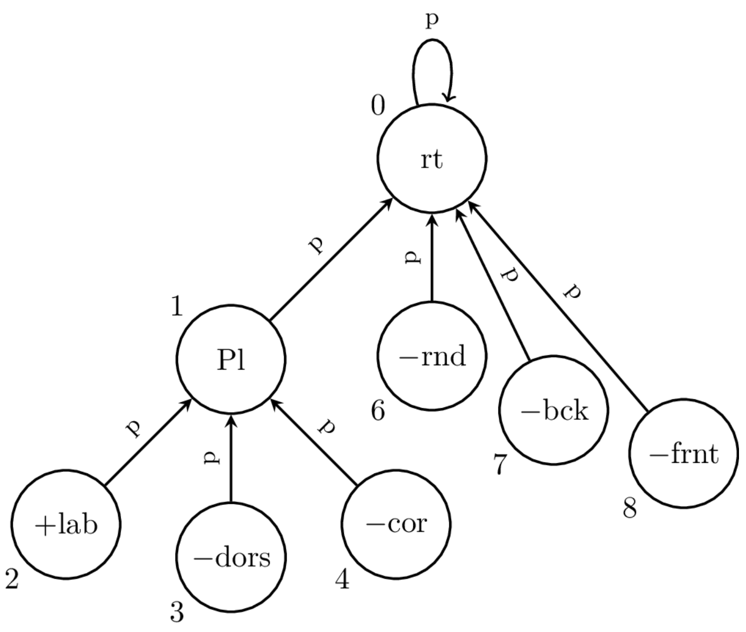


comparing theories: **v-features** model

$D = \{0, 1, 2, 3, 4, 6, 7, 8\}$
$P_{rt} = \{0\}$
$P_{Pl} = \{1\}$
$P_{+lab} = \{2\}$
$P_{-dors} = \{3\}$
$P_{-cor} = \{4\}$
$P_{-rnd} = \{6\}$
$P_{-bck} = \{7\}$
$P_{-frnt} = \{8\}$

$$parent(x) = \begin{cases} 0 \Leftrightarrow x \in \{0,1,6,7,8\} \\ 1 \Leftrightarrow x = \{2,3,4\} \end{cases}$$

v-features
/p/



the following slides provide a transduction in quantifier-free first-order logic (QF) that translates between the unified model and the v-features model

the transduction: unified → v-features

$$
\begin{aligned}
\text{rt}(x^1) &\stackrel{\text{def}}{=} \text{rt}(x)\\
\text{Place}(x^1) &\stackrel{\text{def}}{=} \text{C-place}(x)\\
+\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{labial}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
+\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{coronal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
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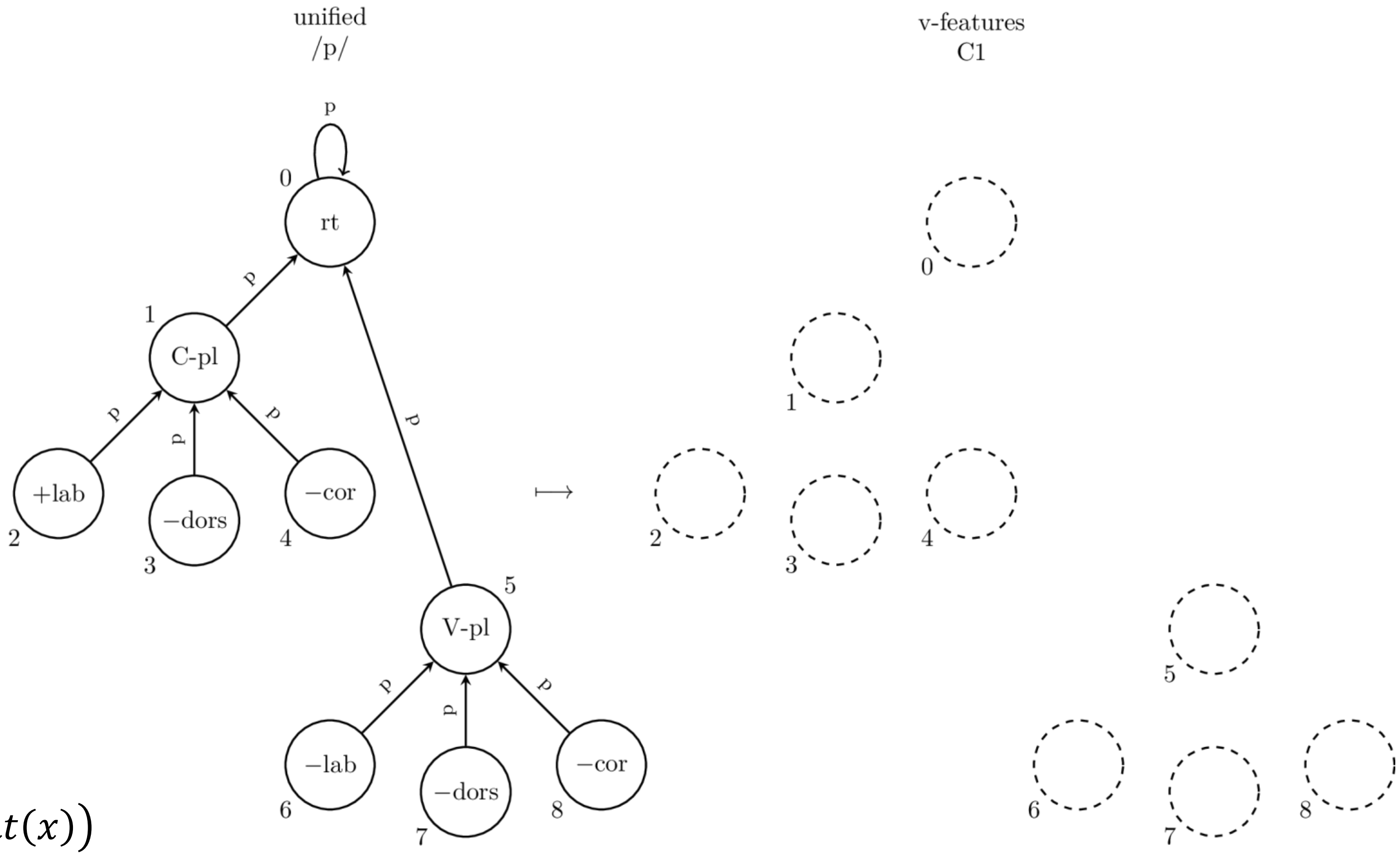
\end{aligned}
$$



the transduction: unified ⟶ v-features

$$\mathrm{rt}(x^1) \overset{\text{def}}{=} \mathrm{rt}(x)$$
$$\mathrm{Place}(x^1) \overset{\text{def}}{=} \text{C-place}(x)$$
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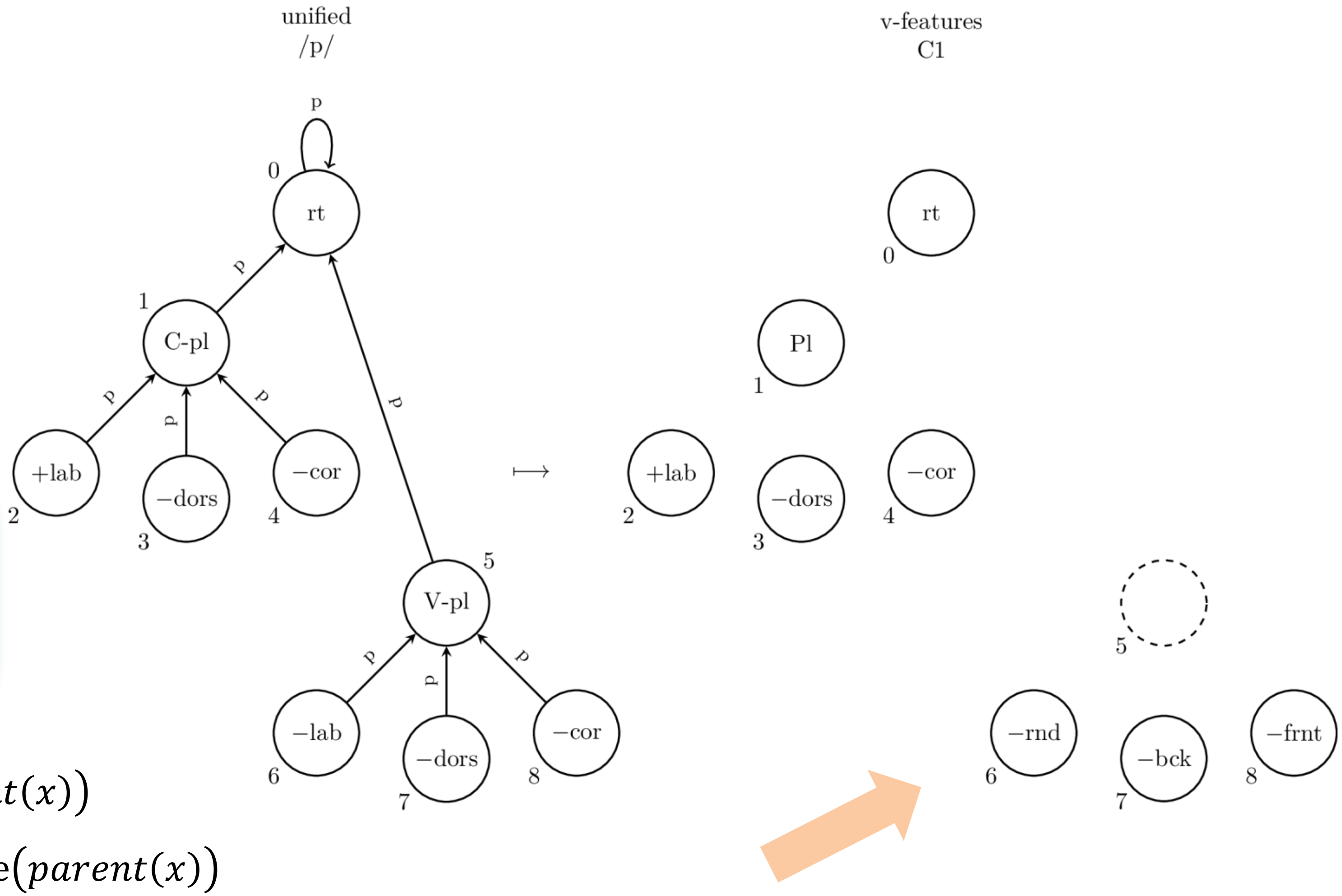

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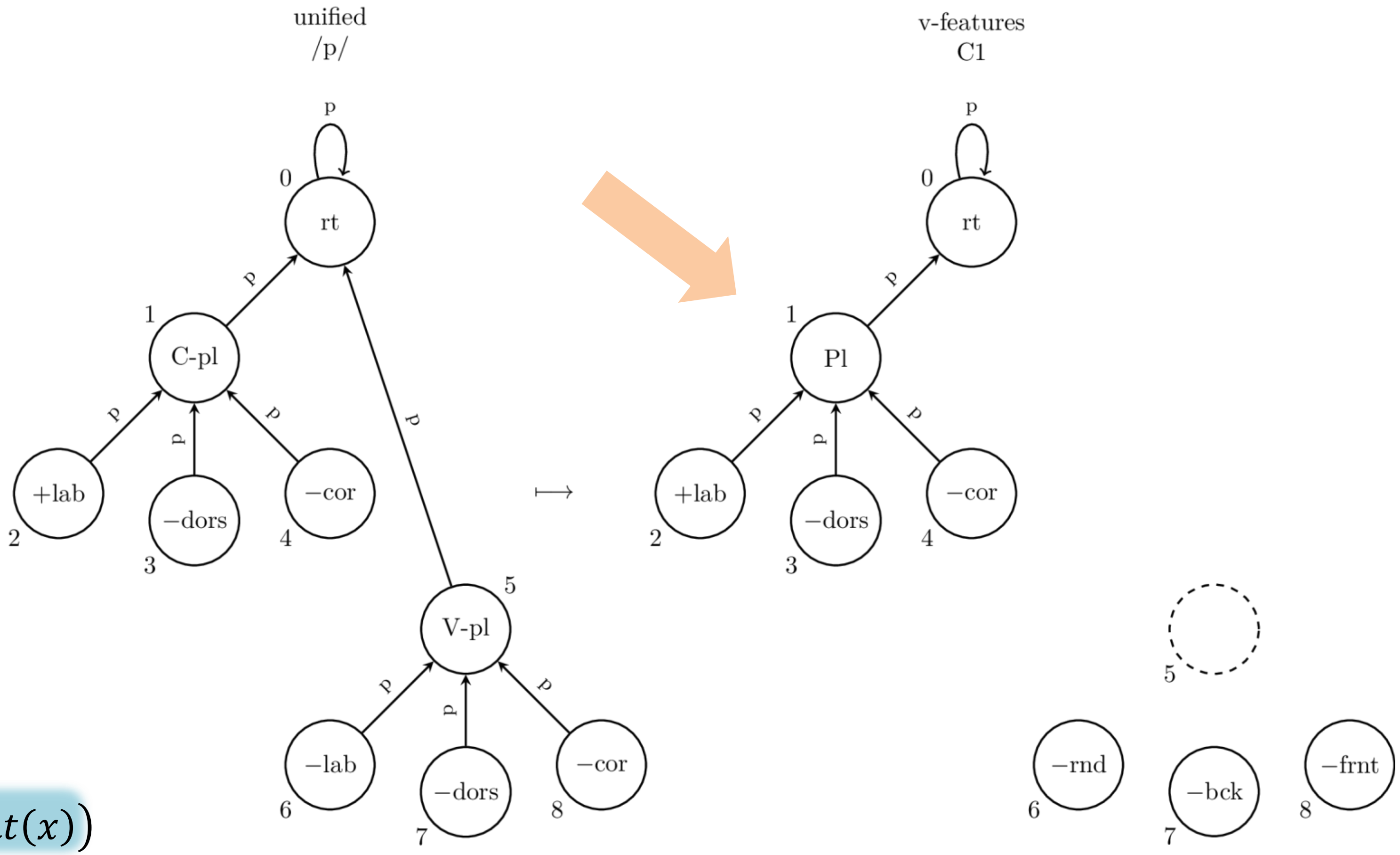

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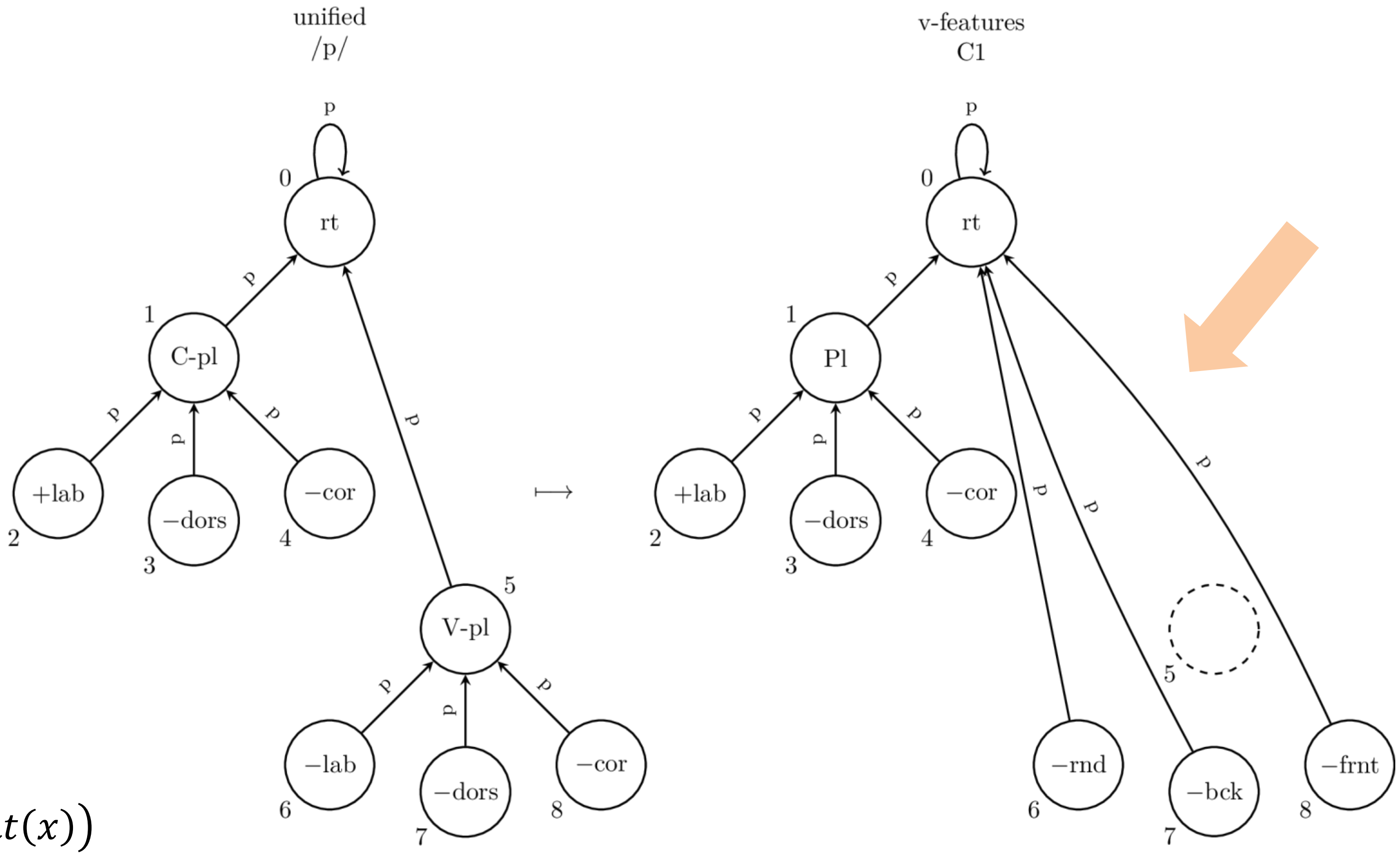
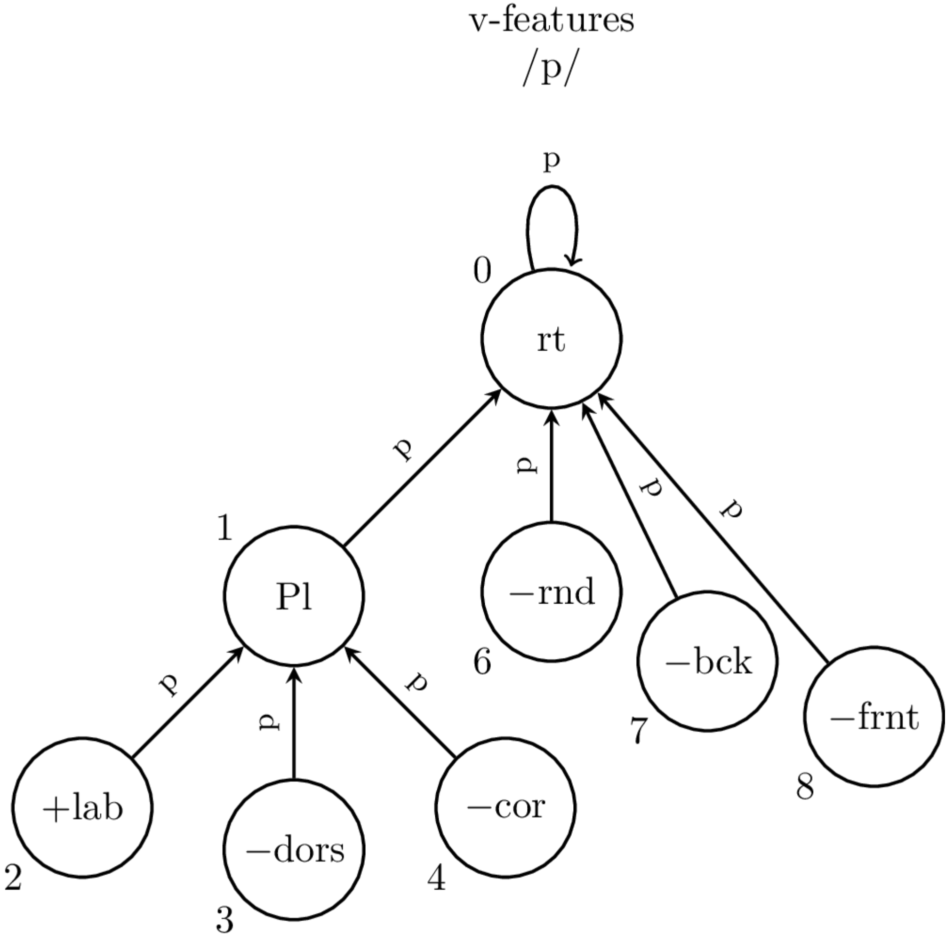
+\mathrm{front}(x^1) &\overset{\mathrm{def}}{=} +\mathrm{coronal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
+\mathrm{back}(x^1) &\overset{\mathrm{def}}{=} +\mathrm{dorsal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\mathrm{round}(x^1) &\overset{\mathrm{def}}{=} -\mathrm{labial}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\mathrm{front}(x^1) &\overset{\mathrm{def}}{=} -\mathrm{coronal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\mathrm{back}(x^1) &\overset{\mathrm{def}}{=} -\mathrm{dorsal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
\mathit{parent}(x^1) &\overset{\mathrm{def}}{=} \begin{cases} \big(\mathit{parent}(x)\big)^1 \Leftrightarrow \neg\text{V-place}\big(\mathit{parent}(x)\big)\\ \Big(\mathit{parent}\big(\mathit{parent}(x)\big)\Big)^1 \Leftrightarrow \text{V-place}\big(\mathit{parent}(x)\big) \end{cases}
\end{aligned}
$$



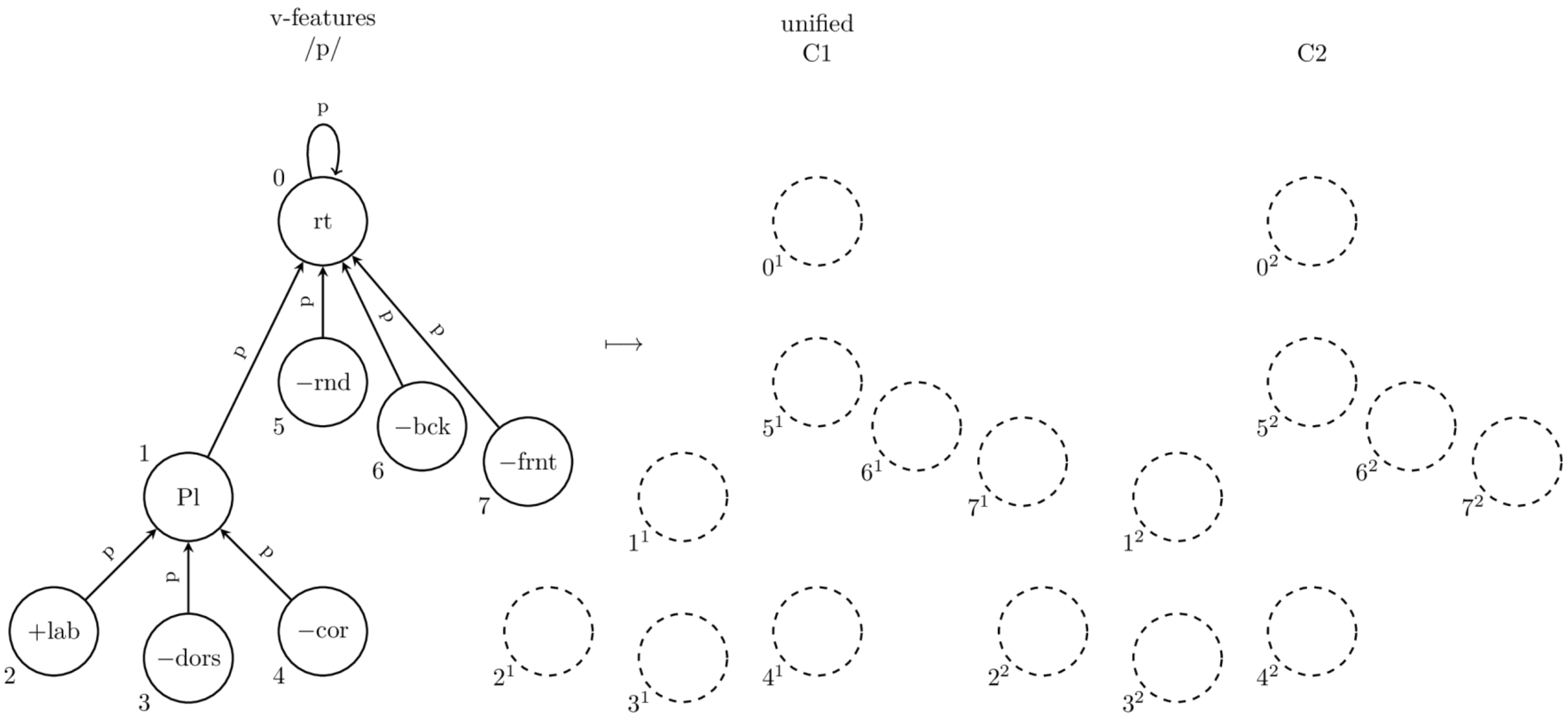
the transduction: unified ⟶ v-features

$$
\begin{aligned}
\text{rt}(x^1) &\overset{\text{def}}{=} \text{rt}(x)\\
\text{Place}(x^1) &\overset{\text{def}}{=} \text{C-place}(x)\\
+\text{labial}(x^1) &\overset{\text{def}}{=} +\text{labial}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
+\text{coronal}(x^1) &\overset{\text{def}}{=} +\text{coronal}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
+\text{dorsal}(x^1) &\overset{\text{def}}{=} +\text{dorsal}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
-\text{labial}(x^1) &\overset{\text{def}}{=} -\text{labial}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
-\text{coronal}(x^1) &\overset{\text{def}}{=} -\text{coronal}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
-\text{dorsal}(x^1) &\overset{\text{def}}{=} -\text{dorsal}(x) \wedge \text{C-place}(\mathit{parent}(x))\\
+\text{round}(x^1) &\overset{\text{def}}{=} +\text{labial}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
+\text{front}(x^1) &\overset{\text{def}}{=} +\text{coronal}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
+\text{back}(x^1) &\overset{\text{def}}{=} +\text{dorsal}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
-\text{round}(x^1) &\overset{\text{def}}{=} -\text{labial}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
-\text{front}(x^1) &\overset{\text{def}}{=} -\text{coronal}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
-\text{back}(x^1) &\overset{\text{def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(\mathit{parent}(x))\\
\mathit{parent}(x^1) &\overset{\text{def}}{=} \begin{cases} \left(\mathit{parent}(x)\right)^1 \Leftrightarrow \neg\text{V-place}(\mathit{parent}(x)) \\ \left(\mathit{parent}(\mathit{parent}(x))\right)^1 \Leftrightarrow \text{V-place}(\mathit{parent}(x)) \end{cases}
\end{aligned}
$$



final output structure, rearranged with unlicensed nodes deleted

the transduction: v-features ⟶ unified



$$\mathrm{rt}(x^1) \overset{\mathrm{def}}{=} \mathrm{rt}(x)$$

$$+\mathrm{labial}(x^1) \overset{\mathrm{def}}{=} +\mathrm{round}(x) \lor \mathrm{labial}(x)$$

$$+\mathrm{coronal}(x^1) \overset{\mathrm{def}}{=} +\mathrm{front}(x) \lor \mathrm{coronal}(x)$$

$$+\mathrm{dorsal}(x^1) \overset{\mathrm{def}}{=} +\mathrm{back}(x) \lor \mathrm{coronal}(x)$$

$$-\mathrm{labial}(x^1) \overset{\mathrm{def}}{=} -\mathrm{round}(x) \lor \mathrm{labial}(x)$$

$$-\mathrm{coronal}(x^1) \overset{\mathrm{def}}{=} -\mathrm{front}(x) \lor \mathrm{coronal}(x)$$

$$-\mathrm{dorsal}(x^1) \overset{\mathrm{def}}{=} -\mathrm{back}(x) \lor \mathrm{coronal}(x)$$

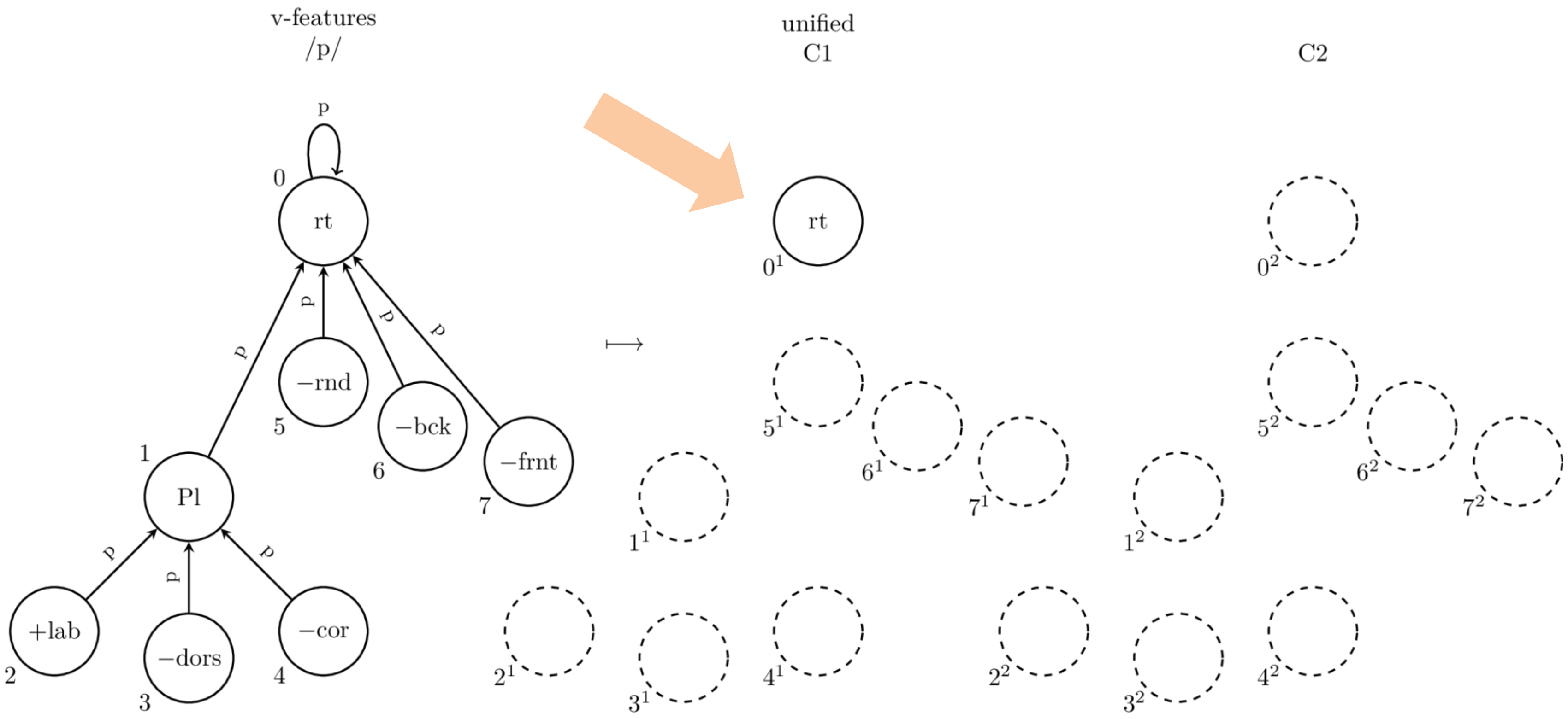
$$\mathrm{C\text{-}place}(x^1) \overset{\mathrm{def}}{=} \mathrm{Place}(x)$$

$$\mathrm{V\text{-}place}(x^2) \overset{\mathrm{def}}{=} \mathrm{rt}(x)$$

$$parent(x^1) \overset{\mathrm{def}}{=} \begin{cases} \left(parent(x)\right)^1 \Leftrightarrow \neg\mathsf{vowelFeature}(x)) \\ \left(parent(x)\right)^2 \Leftrightarrow \mathsf{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \overset{\mathrm{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

the transduction: v-features ⟶ unified



$$\mathrm{rt}(x^1) \overset{\text{def}}{=} \mathrm{rt}(x)$$
$$+\mathrm{labial}(x^1) \overset{\text{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)$$
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$$\text{C-place}(x^1) \overset{\text{def}}{=} \mathrm{Place}(x)$$
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$$parent(x^2) \overset{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

the transduction: v-features ⟶ unified

$$\mathrm{rt}(x^1) \overset{\text{def}}{=} \mathrm{rt}(x)$$
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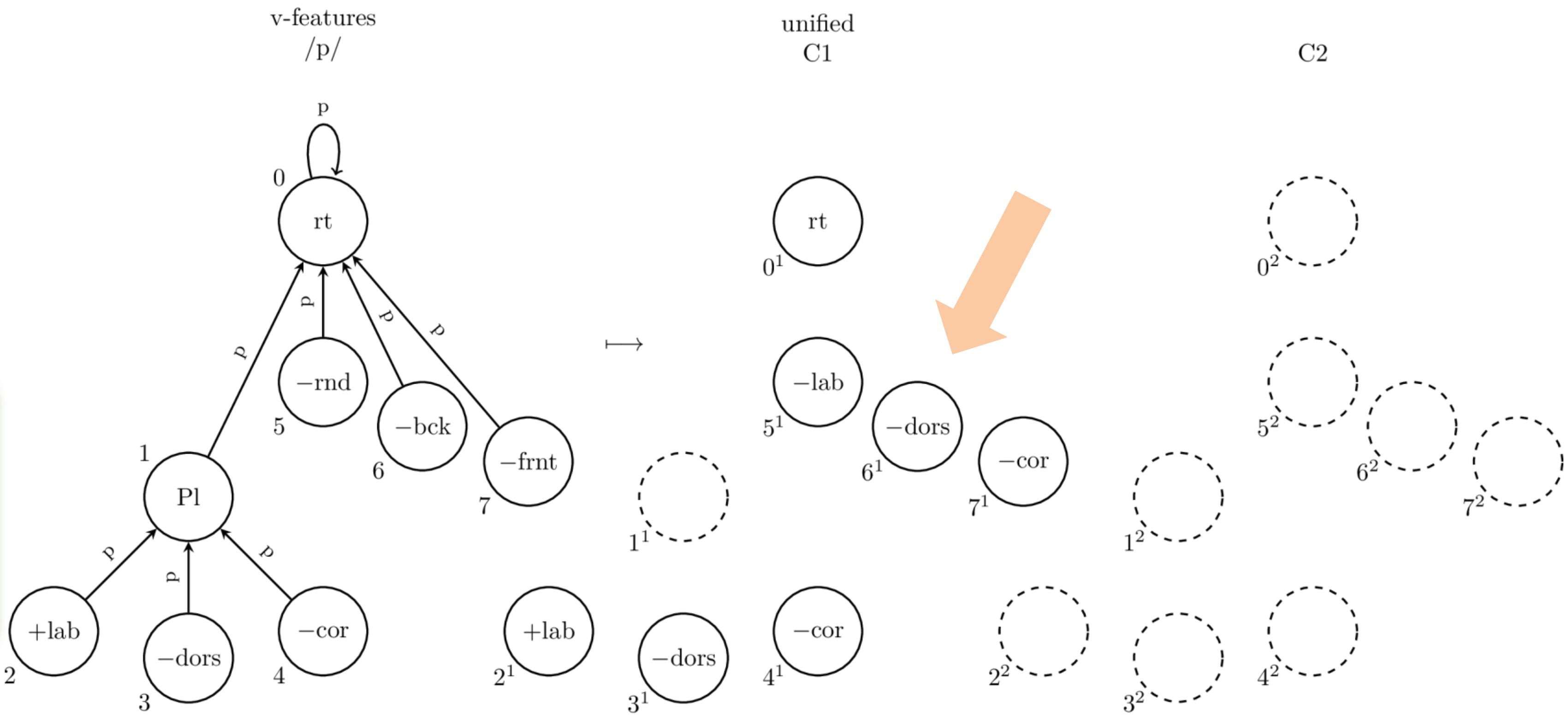
$$parent(x^2) \overset{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$



the transduction: v-features ⟶ unified

$$\text{rt}(x^1) \overset{\text{def}}{=} \text{rt}(x)$$
$$+\text{labial}(x^1) \overset{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x)$$
$$+\text{coronal}(x^1) \overset{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x)$$
$$+\text{dorsal}(x^1) \overset{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x)$$
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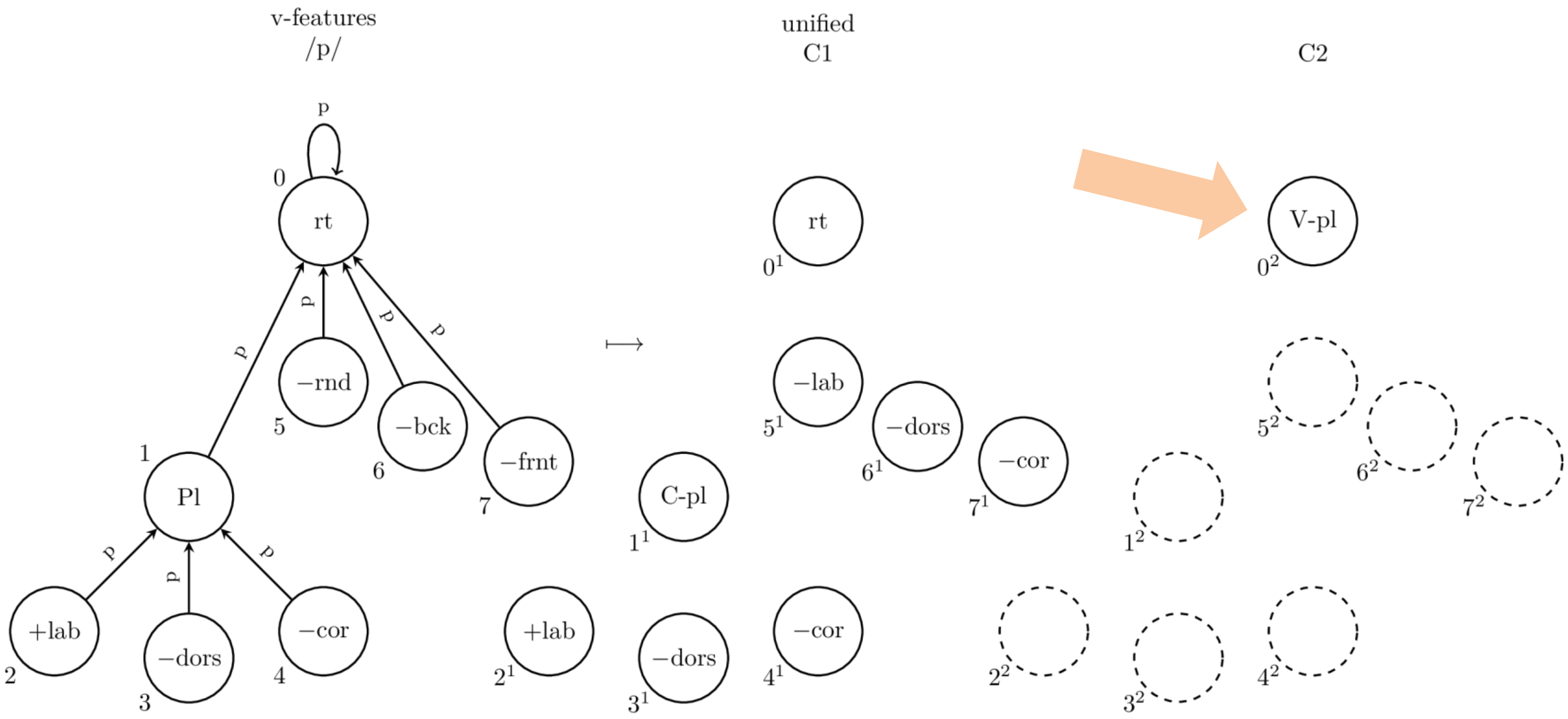
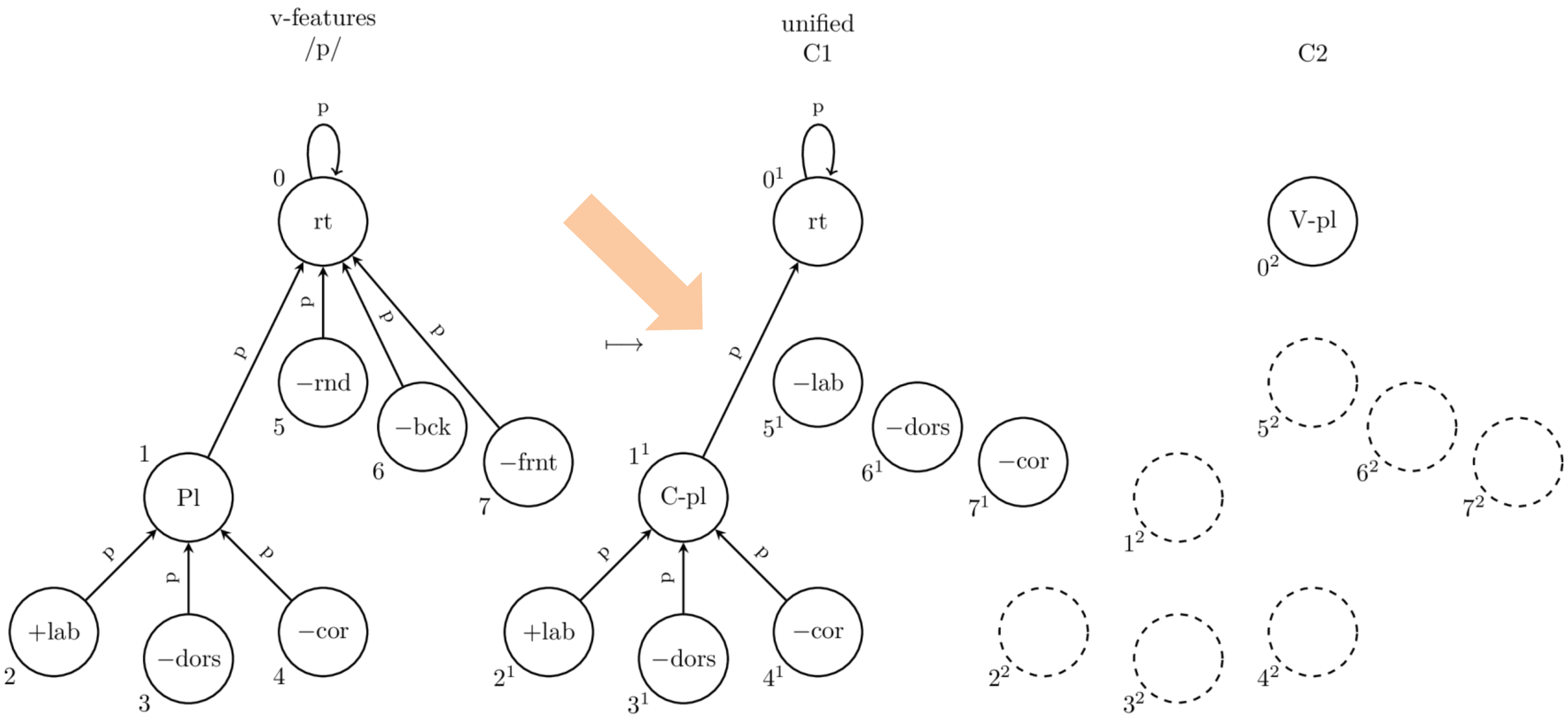
$$parent(x^2) \overset{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)$$



the transduction: v-features ⟶ unified



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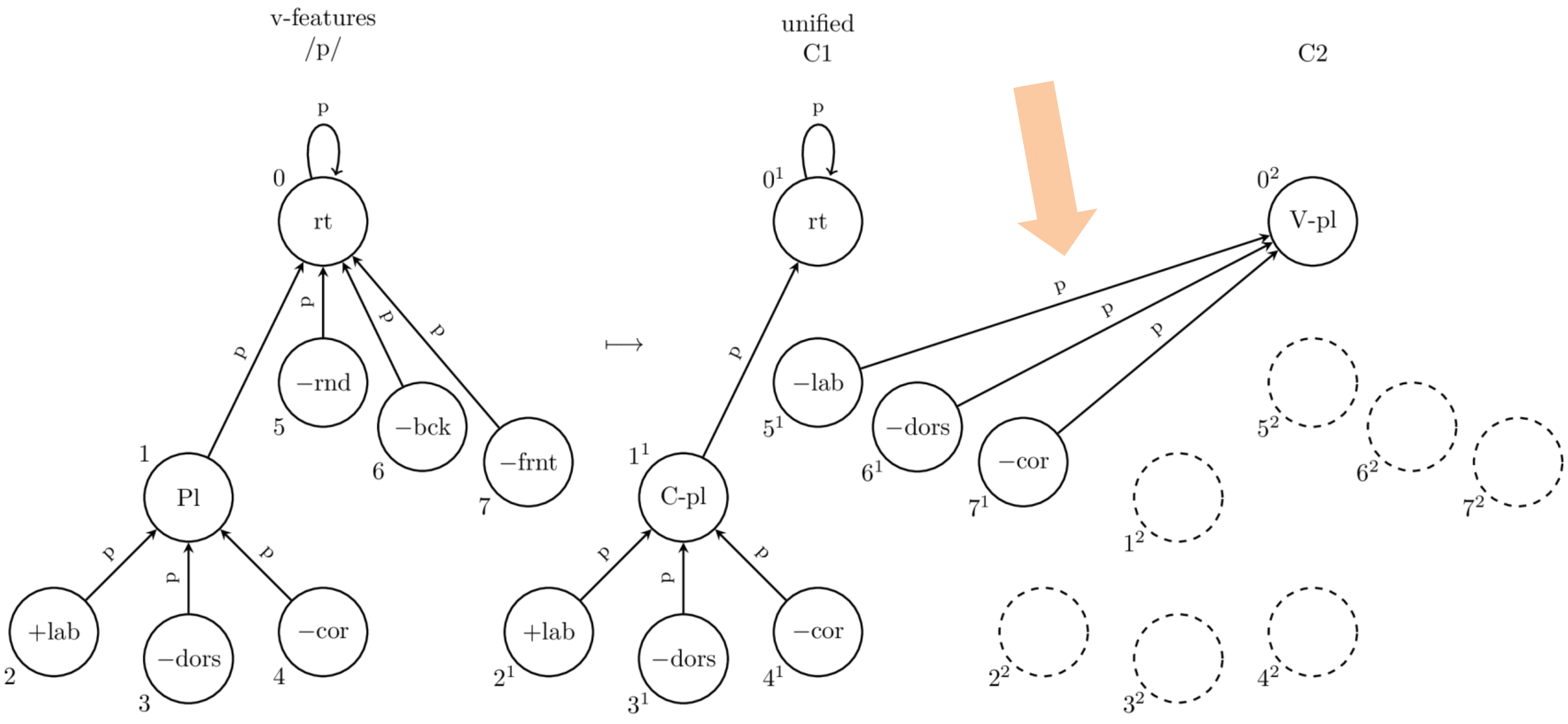
$$parent(x^2) \overset{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \vee -\mathrm{round}(x) \vee +\mathrm{front}(x) \vee -\mathrm{front}(x) \vee +\mathrm{back}(x) \vee -\mathrm{back}(x)$$

the transduction: v-features ⟶ unified



$$\mathrm{rt}(x^1) \stackrel{\text{def}}{=} \mathrm{rt}(x)$$

$$+\mathrm{labial}(x^1) \stackrel{\text{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)$$

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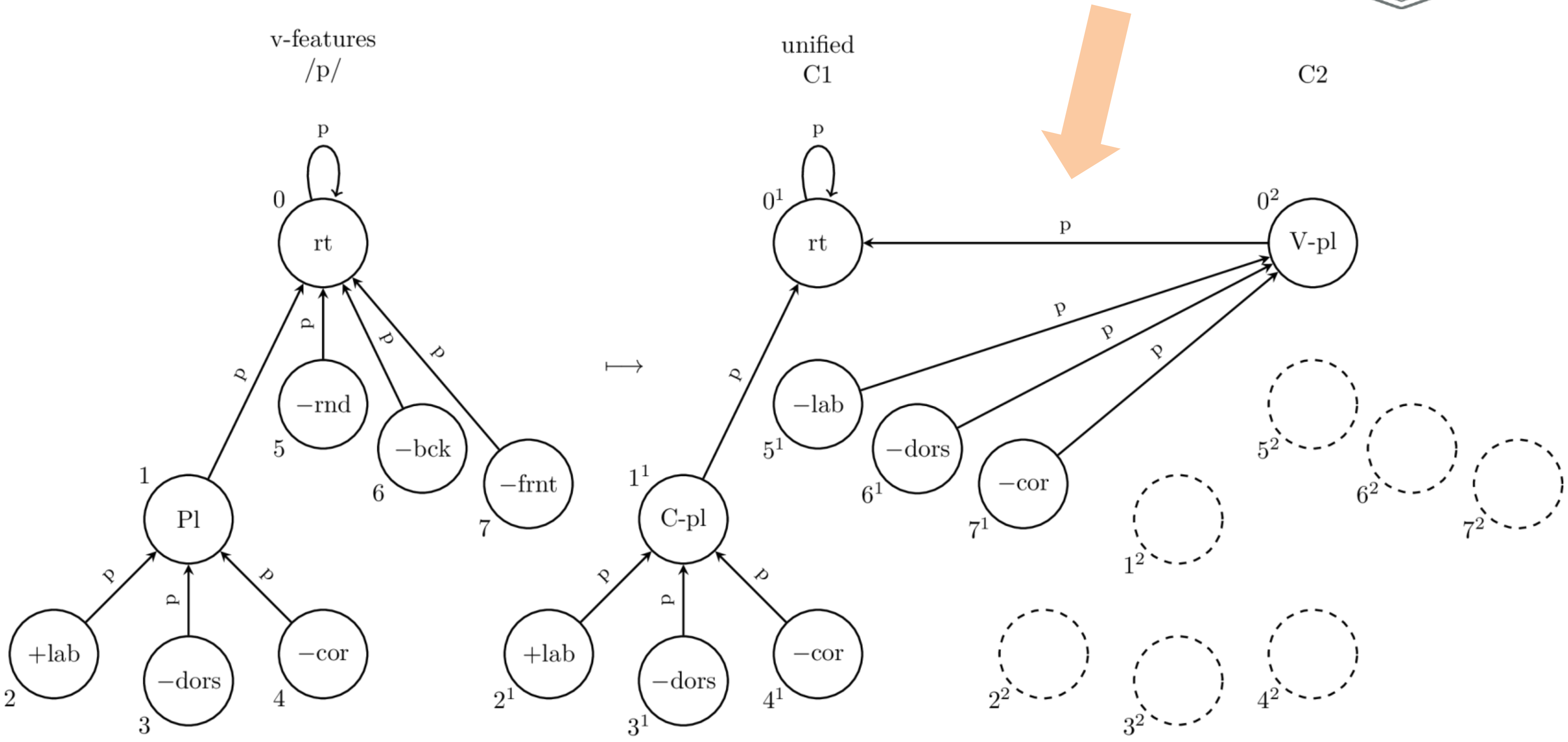
$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \vee -\mathrm{round}(x) \vee +\mathrm{front}(x) \vee -\mathrm{front}(x) \vee +\mathrm{back}(x) \vee -\mathrm{back}(x)$$

the transduction: v-features ⟶ unified



$$
\begin{aligned}
\mathrm{rt}(x^1) &\stackrel{\text{def}}{=} \mathrm{rt}(x)\\
+\mathrm{labial}(x^1) &\stackrel{\text{def}}{=} +\mathrm{round}(x) \lor \mathrm{labial}(x)\\
+\mathrm{coronal}(x^1) &\stackrel{\text{def}}{=} +\mathrm{front}(x) \lor \mathrm{coronal}(x)\\
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parent(x^1) &\stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\mathrm{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}\\
parent(x^2) &\stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)
\end{aligned}
$$

$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \lor -\mathrm{round}(x) \lor +\mathrm{front}(x) \lor -\mathrm{front}(x) \lor +\mathrm{back}(x) \lor -\mathrm{back}(x)$$

unified and v-features are QF-bi-interpretable

and are therefore notational variants?

"The paper capitalises on structural similarities apparent in the Yip and Bao models to show that one can be freely translated into another, and *vice versa*. Such a translation does not result in any loss of the contrasts expressible by either theory. **Given these two results, the main claim of the paper is that the two representational proposals do not constitute distinct theories, but are notationally equivalent.**"

(Oakden 2021: 258)

"A QF transduction is extremely restricted in the degree to which the output can differ from the input because QF is a weak logical language limited to local operations. **QF-bi-interpretability can therefore be considered an indication of notational equivalence.**"

(Strother-Garcia 2019: 39)

enumerating natural class extensions

the full range of contrasts considered:

labialized consonants *velarized consonants* *rounded vowels*

{p, t, k, p^{w}, t^{w}, k^{w}, p^{j}, t^{j}, k^{j}, $p^{ɣ}$, $t^{ɣ}$, $k^{ɣ}$, kp, tp, kt, y, ʉ, u, i, ɨ, ɯ}

plain consonants *palatalized consonants* *double articulations* *unrounded vowels*

given this set of contrasts, how do the natural class extensions of **unified** compare with those of **v-features**?

- **by design**, unified and v-features do not predict the same natural classes
- but we'll look anyway

enumerating natural class extensions

general procedure: (e.g. how I currently have it programmed in python)

initialize **natural_classes** as dict

- for each licit structure **s** in some theory **T**: *collect factors*
  - for each factor (connected substructure) **f** of **s**:
    - add **f** to **natural_classes** if f not in **natural_classes**
- for each factor **f** in **natural_classes**: *collect segments with each factor*
  - for each segment **s**
    - if f is a factor of **s**:
      - add **s** to **natural_classes**[**f**]

comparing natural class extensions

define each theory as a set of graphs, where each graph is a possible segment

https://github.com/nickdanis/autosegx/

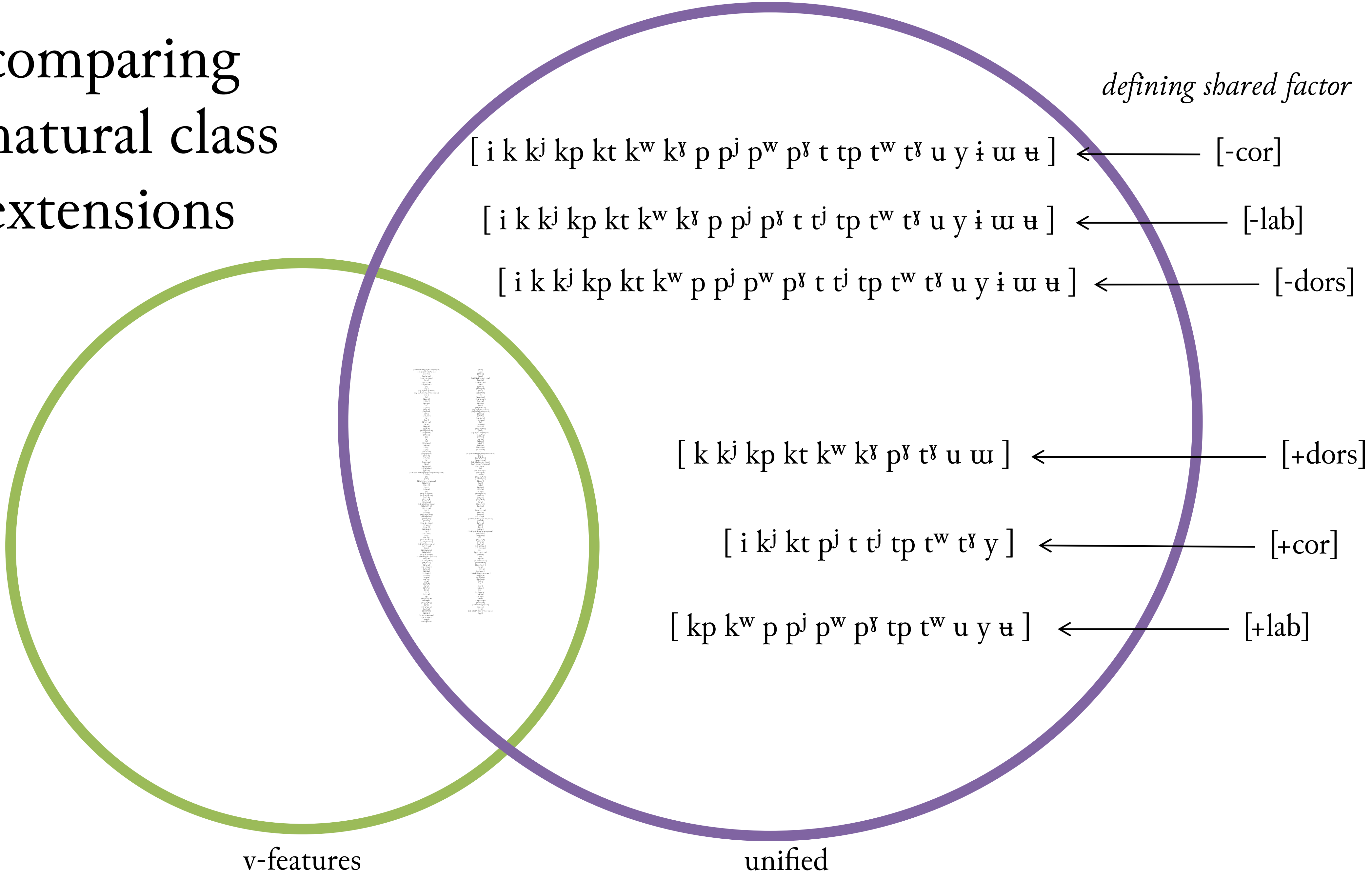
```
The theories are NOT natural class preserving.

6 natural class(es) unique to unified.
        1. [ kp kw p pj pw pɣ tp tw u y ʉ ]
                Defining factors:
                (1) +lab
        2. [ i k kj kp kt kw kɣ p pj pɣ t tj tp tw tɣ u y ɨ ɯ ʉ ]
                Defining factors:
                (1) -lab
        3. [ i kj kt pj t tj tp tw tɣ y ]
                Defining factors:
                (1) +cor
        4. [ k kj kp kt kw kɣ pɣ tɣ u ɯ ]
                Defining factors:
                (1) +dors
        5. [ i k kj kp kt kw kɣ p pj pw pɣ t tp tw tɣ u y ɨ ɯ ʉ ]
                Defining factors:
                (1) -cor
        6. [ i k kj kp kt kw p pj pw pɣ t tj tp tw tɣ u y ɨ ɯ ʉ ]
                Defining factors:
                (1) -dors

0 natural class(es) unique to v-features.

238 natural class(es) shared.
```

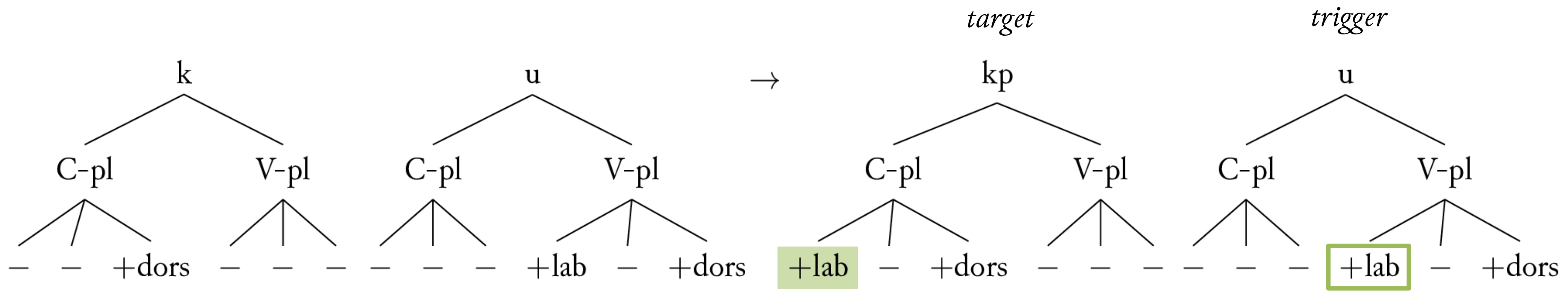
comparing natural class extensions



ku → kpu

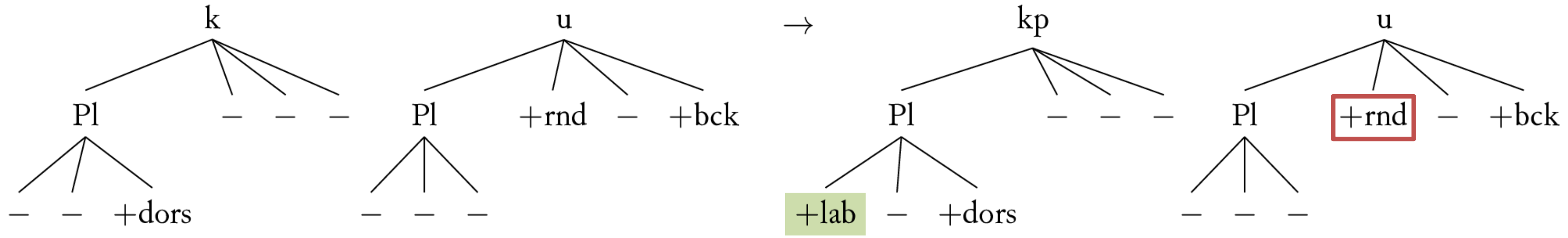
Sharing is Caring 🤝
the target and trigger of an assimilation process should form a natural class *as a result of that assimilation process*

unified



structural change results in **new** natural class
this is an assimilation process

v-features



structural change does **not** result in new natural class
this is not an assimilation process

one process or two

imagine a language that prohibits the following consonants:

[p, p^{w}, t^{w}, k^{w}, p^{j}, p^{ɣ}, kp, tp, y, ʉ, u]

- in unified:
  1. *[+labial] targets entire set above

- in v-features:
  1. *[+labial] targets only [p, p^{w}, p^{j}, p^{ɣ}, kp, tp], *and*
  2. *[+round] targets only [t^{w}, k^{w}, y, ʉ, u]

if the same pattern must be analyzed as **two** different processes/constraints in one theory versus as **one** process/constraint in another theory, **this makes different empirical predictions**

comparing theories

(1) *Conditions for notational equivalence*

a. Two models do not differ in their empirical predictions.

b. Two models represent the same set of abstract properties, differing only superficially.

(from Oakden 2021, summarizing Fromkin 2010)

- if we take seriously assumptions like sharing is caring 🤝, **then a QF-bi-interpretable contrast-preserving transduction is not enough to satisfy (1a) above**
  (and arguably 1b is not satisfied if predicted natural classes are in these sets of properties)

(some) existing transductions

transduction	logic	contrast preserving	natural class preserving
unified vs. v-features	QF	yes	no
Oakden (2021)	QF	yes	no
Danis & Jardine (2019)	FO	yes*	??
Cahill & Parkinson (1997)**	QF	yes	yes

* the segments in the transduction are those that were optima in Shih & Inkelas (2019), but the general set of contrasts are likely distinct

** this was not given as a transduction proper, but it is simple to construct one from their claim

Oakden (2021) & tonal geometry

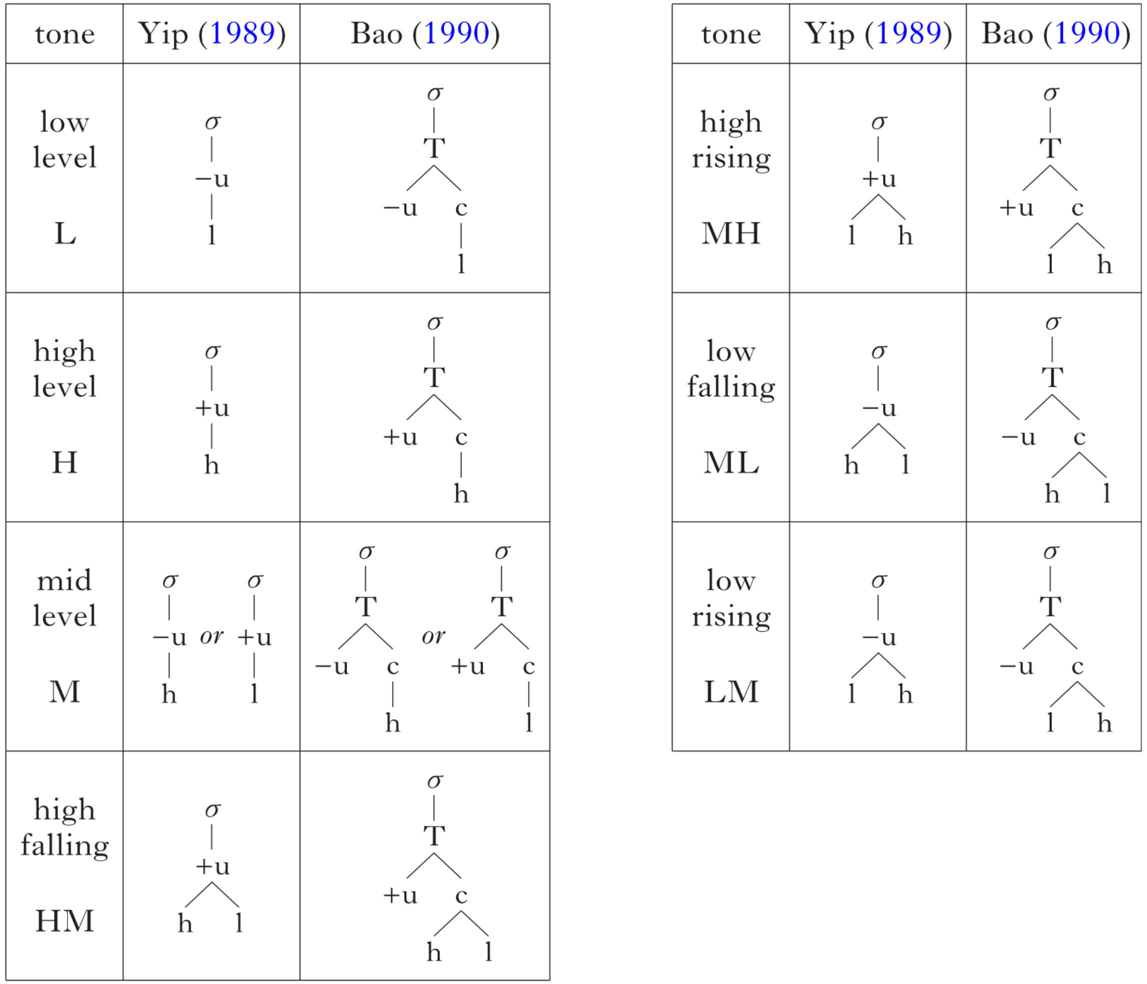
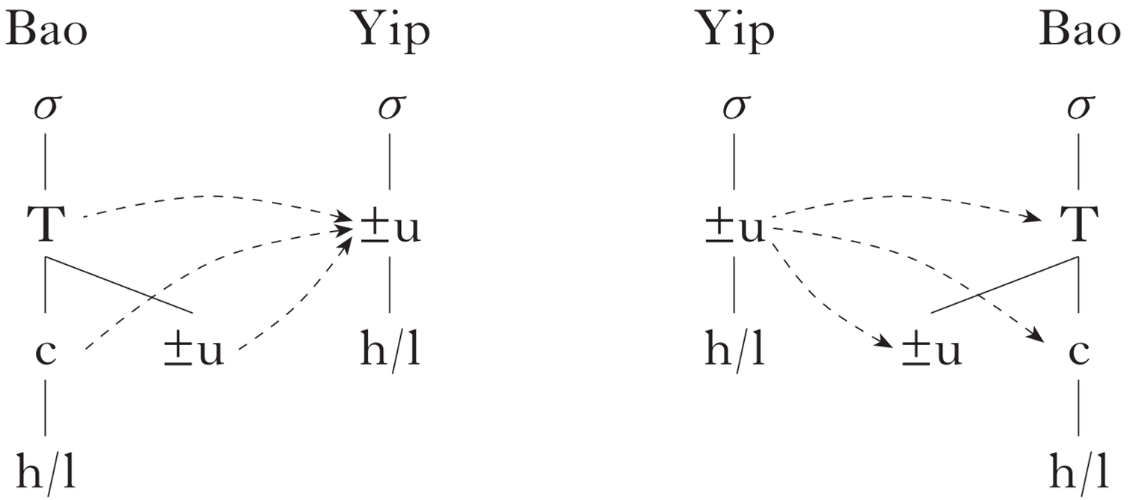


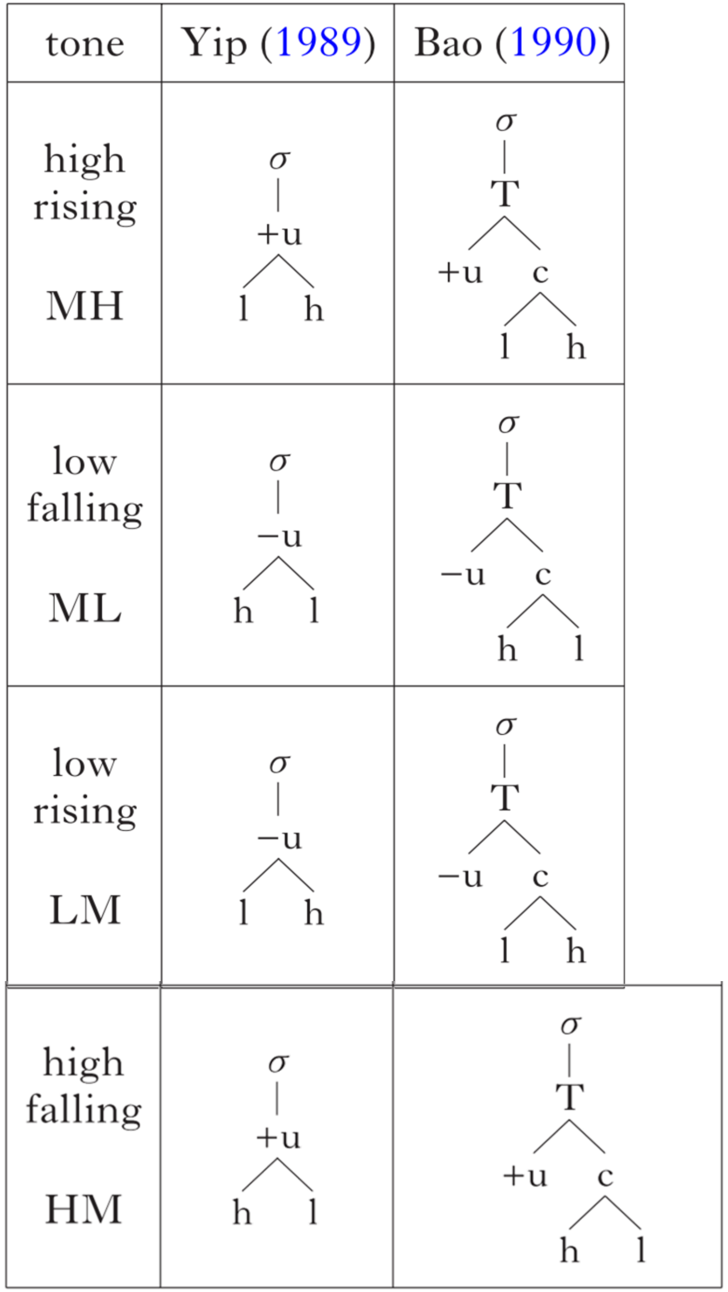
Table I
Level and contour tonal contrasts in Yip (1989) and Bao (1990).



- Oakden (2021) provides a non-size-preserving QF transduction (above) between two theories of tone sandhi (left), arguing for notational equivalence
- **is this transduction also natural class preserving?**

Oakden (2021)

- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's



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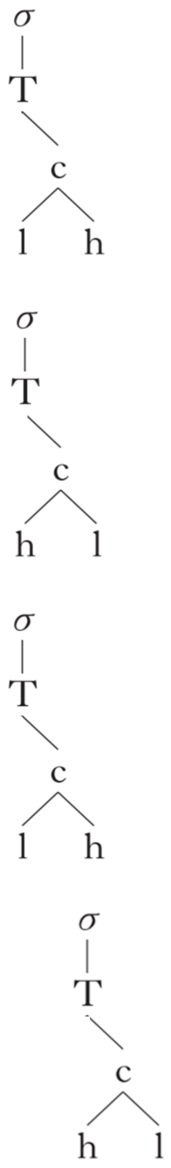


introduction
definitions
example transduction
existing transductions
broader significance
conclusion

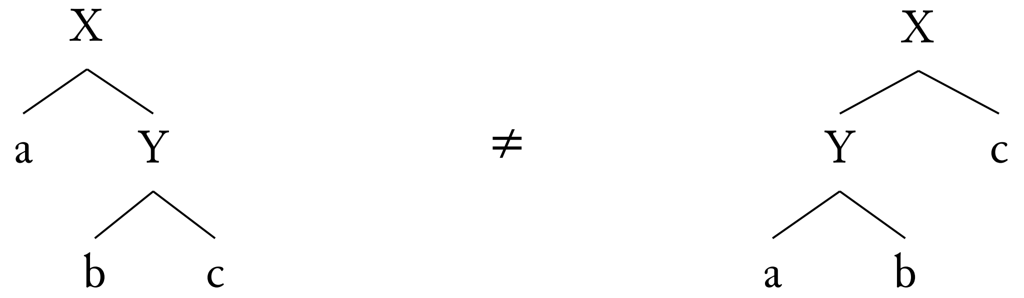
strong generative capacity for **syntax**

" The study of strong generative capacity is related to the study of descriptive adequacy, in the sense defined. A grammar is descriptively adequate if it strongly generates the correct set of structural descriptions. A theory is descriptively adequate if its strong generative capacity includes the system of structural descriptions for each natural language; otherwise, it is descriptively inadequate."

(Chomsky 1969: 60)

strong generative capacity for **syntax**

structural description:



surface string: abc = abc

surface strings are identical (weak equivalence), but the constituency structures do not match (no strong equivalence)

strong generative capacity

- ...in syntax:
  - Chomsky's definition often criticized (see Miller 1999 and references therein)
  - Miller (1999) reworks definition of SGC for syntax in robust model theory
- ...in morphology:
  - Dolatian et al. (2021) define and show divergence of WGC and SGC for various morphological processes and their transductions
- ...in phonology:
  - "In morphology and phonology, there are fewer debates on generative capacity. We speculate that this is due to two issues. First, morphology and phonology have comparatively restrictive WGC. Second, it is unclear what external basis (grounding) should be used for SGC, and thus what diagnostics or metrics to use." (Dolatian et al. 2021: 229)

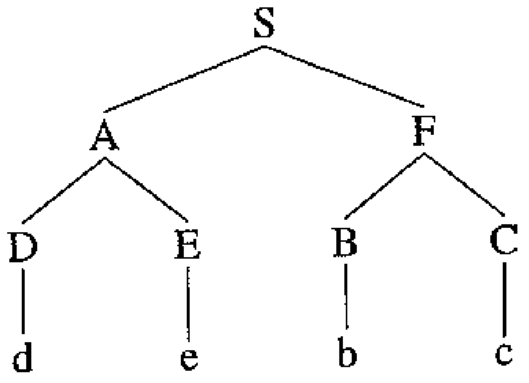
strong generative capacity

"In order to characterize the SGC of a formalism in semantical terms, we need to be able to specify explicitly what it is that is claimed about a given string when a grammar written in that formalism generates that string with a certain structural description. **That is, we must be able to specify which are the intended interpretations of a structural description in a given formalism.** For this purpose we introduce the notion of *Interpretation Domain*. Interpretation Domains are set up to provide explicit characterizations of linguistically significant properties of sentences, independently of specific formalisms. For explicitness and ease of manipulation, we define them in set-theoretic terms."

(Miller 1999: 9)

strong generative capacity (Miller's Version)

- maps **structural descriptions** to item in relevant **interpretation domain** via interpretation function:



output of some CFG

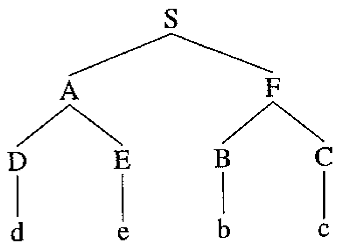
➔

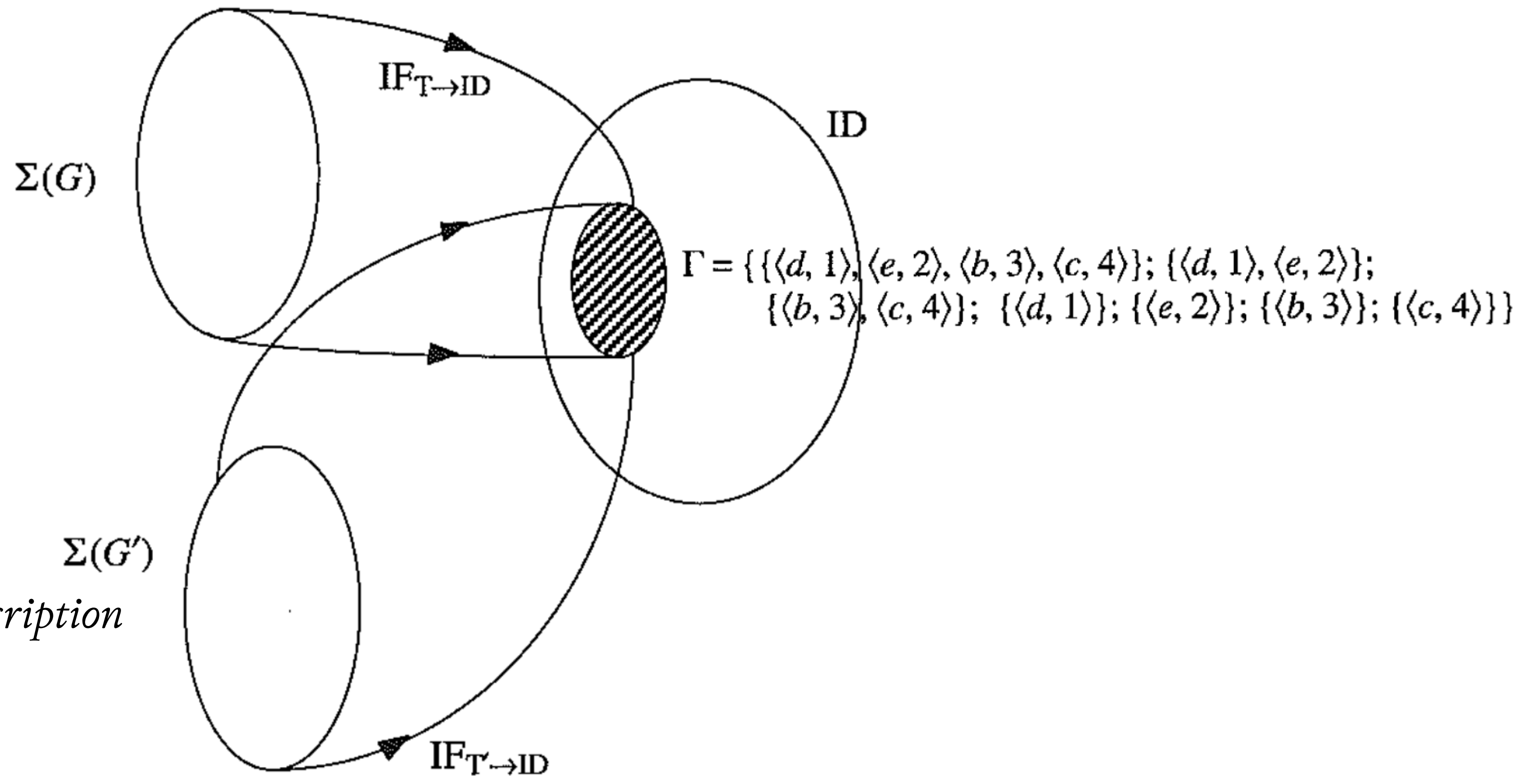
$$\Gamma = \{\{\langle d, 1\rangle, \langle e, 2\rangle, \langle b, 3\rangle, \langle c, 4\rangle\}; \{\langle d, 1\rangle, \langle e, 2\rangle\}; \{\langle b, 3\rangle, \langle c, 4\rangle\}; \{\langle d, 1\rangle\}; \{\langle e, 2\rangle\}; \{\langle b, 3\rangle\}; \{\langle c, 4\rangle\}\}$$

***constituency information** in set theoretic terms*

- if some other structural description, from another theory, also maps to Γ, then these structures (and potentially grammars) are **strongly equivalent with respect to this interpretation domain**

strong generative capacity (Miller's Version)





some other structural description

Figure 3.4. Equivalence of *G* and *G′* in SGC with respect to the Interpretation Domain ID

strong generative capacity…for **phonology**?

- **interpretation domain**: natural classes
- (the set of) structural descriptions of one theory are mapped to the set of natural class extensions in the interpretation domain of natural classes
- if another theory maps to this same set**, the two are strongly equivalent under this interpretation domain**
- *this is exactly the method here*

strong generative capacity…for **phonology**?

- **natural class preservation** should be in the set of diagnostics for evaluating a stronger notion of equivalence for phonological theories
- **contrast preservation** is a weaker notion, entailed by natural class preservation
  - **Proof**: assume two theories are natural class preserving. if they are natural class preserving, they have the same extensions of atomic segments for each class (by def.). if these elements are flattened to a single set for both theories, then $S_1 = S_2$. this is the definition of contrast preserving. therefore the two theories are contrast preserving.
- contrast preservation might be better analog of the WGC

structural descriptions : strings :: natural classes : contrasts

introduction
definitions
example transduction
existing transductions
broader significance
conclusion

takeaways

- a bi-interpretable transduction cannot exist without contrast preservation (conjecture; as there is by definition no longer a one-to-one mapping of structures), but *can* exist without natural class preservation
- the mere *existence* of such a transduction between two theories tells us very little beyond contrast preservation, but the *details* of the transduction rules reflect what, if anything, makes the two theories **linguistically** different

$$+\text{labial}(x^1) \overset{\text{def}}{=} +\text{labial}(x) \vee +\text{round}(x)$$

going forward

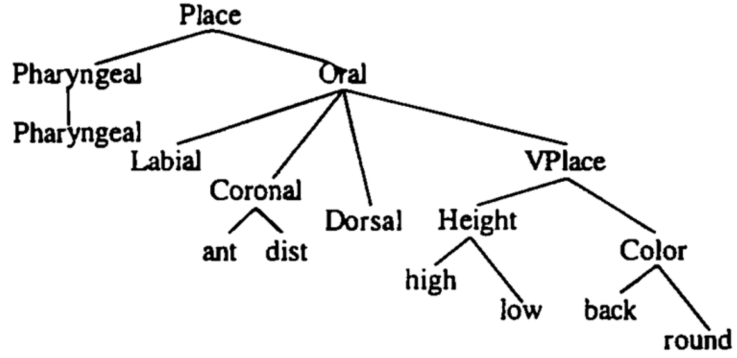
- the definition of *natural class preserving* is based on the representation themselves—can this property be identified by investigating the transduction rules alone?
  - disjunctive labeling
  - loss of labels
- how *else* can transductions themselves be compared and evaluated from a linguistic standpoint?
- **how strongly should our metatheoretical assumptions and expectations about linguistic processes be formalized? (or even said aloud?)**
- and how can these definitions be used for strong and weak equivalence of *mappings*?

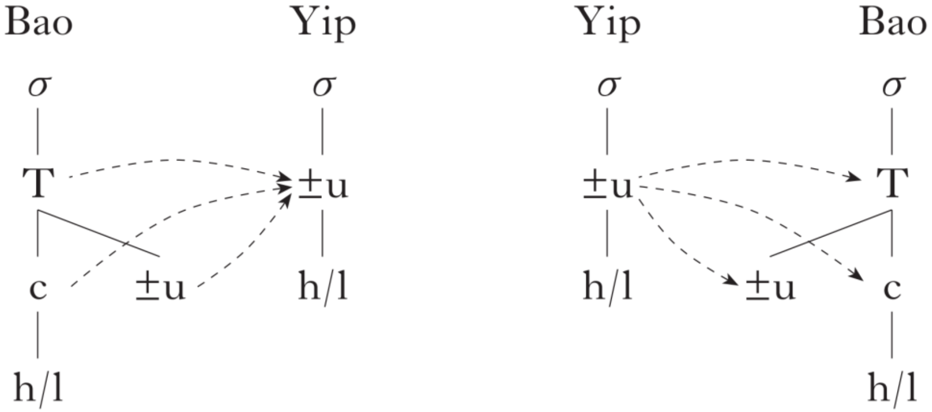
(3) The *Place* class as a set of sets.

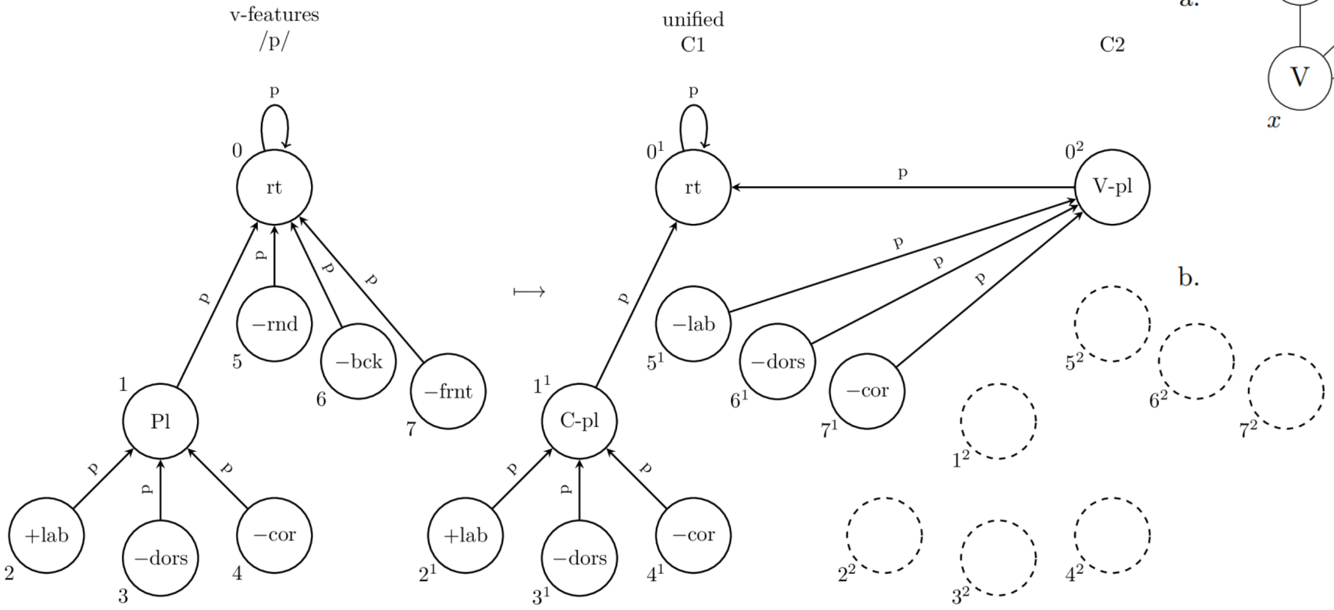
$$\{_{PL}\{_{Ph}\text{ Phary}\}\{_{O}\text{ Lab, Cor, Dor, ant, dist }\{_{VP}\{_{H}\text{ high, low}\}\{_{C}\text{ back, round}\}\}\}\}$$

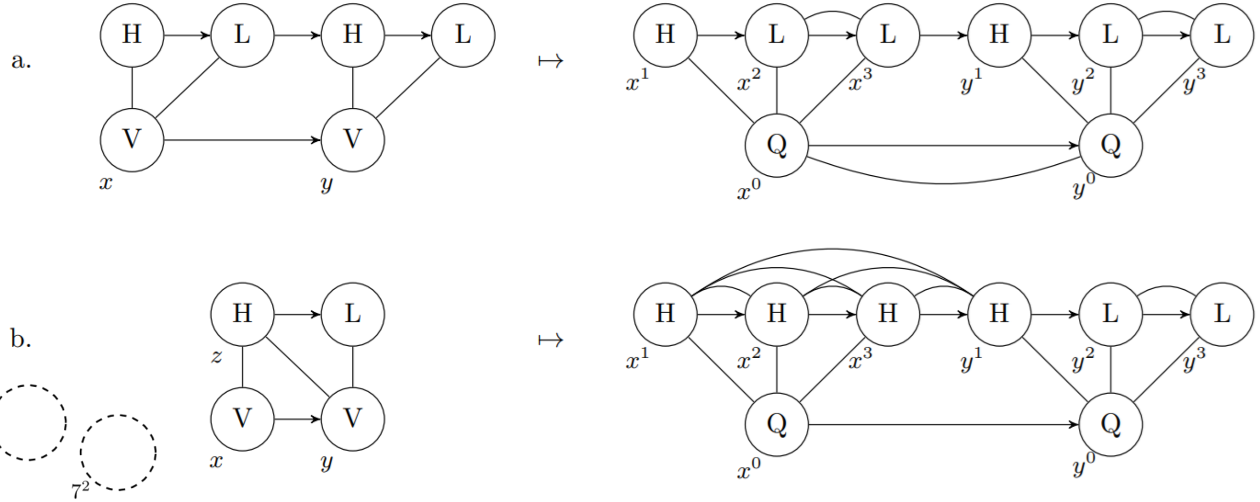
The transition from (3) to (4) is one of notation only.

(4) Feature Geometry (Padgett 1995:398).









natural class preserving

(3) The *Place* class as a set of sets.

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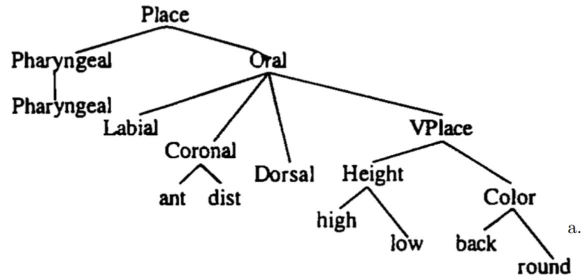
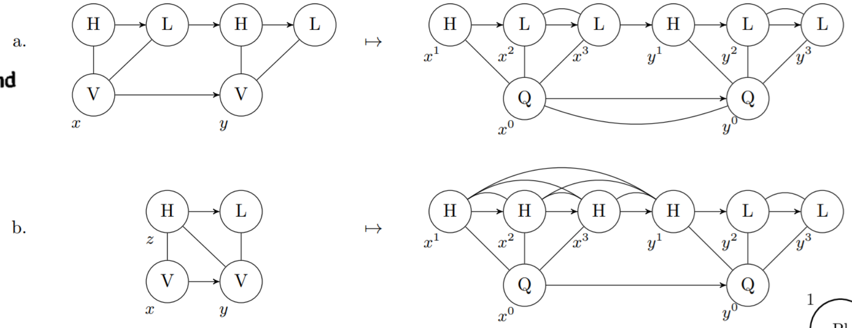
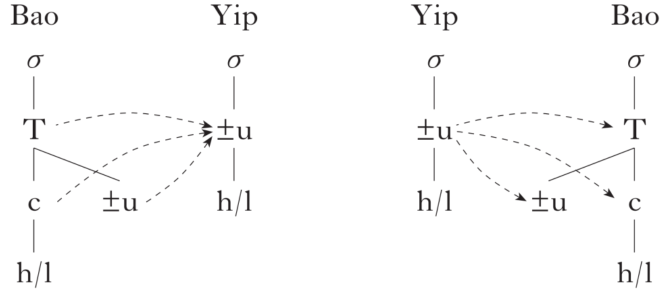
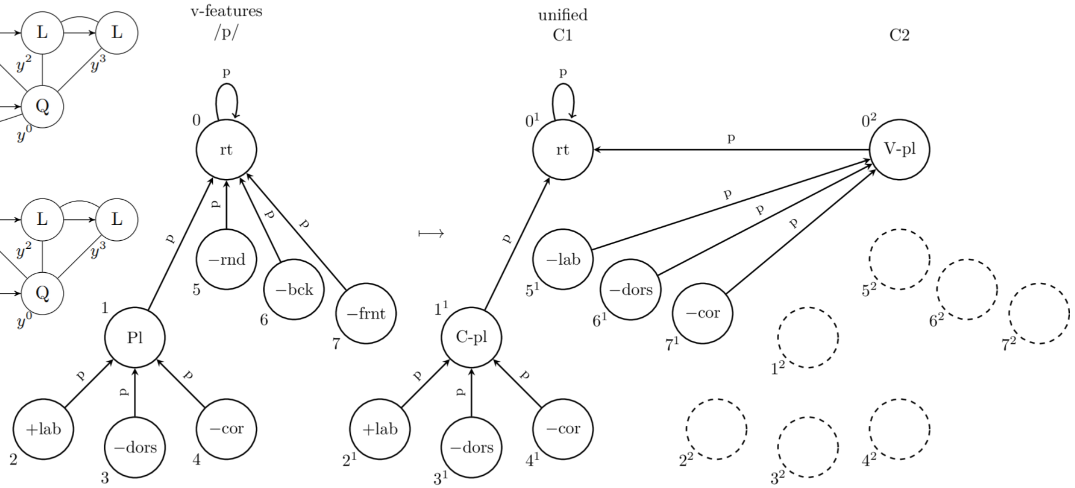




non-natural class preserving





thank you

🗺️ ⟷ 🌎

and thank you Adam Jardine (for first helping with the transductions like 6 years ago);
Jon Rawski & Hosep Dolatian for pointing me towards the SGC;
& previous audiences at Harvard University, Michigan State University, Stony Brook, & Wash U

appendices

strong generative capacity…for **phonology**?

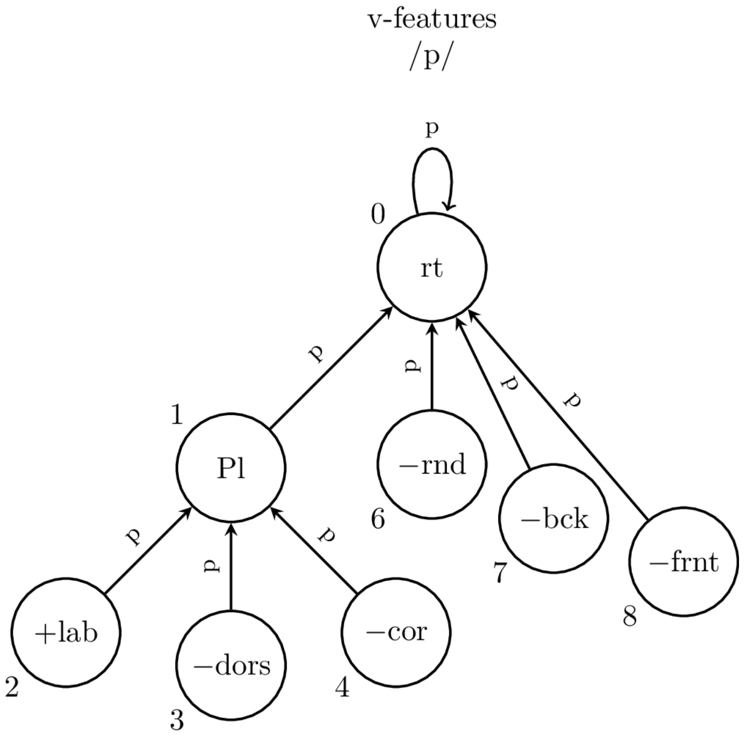
some thoughts **in progress** on **mappings:**

- both models *can* handle a mapping like ku → kpu, **but only one theory treats it as an assimilation process**
- weak equivalence: the same mapping is possible
- strong equivalence: the mapping is the same **type of process** in each model

comparing theories: **v-features** model

$D = \{0, 1, 2, 3, 4, 6, 7, 8\}$
$P_{rt} = \{0\}$
$P_{Pl} = \{1\}$
$P_{+lab} = \{2\}$
$P_{-dors} = \{3\}$
$P_{-cor} = \{4\}$
$P_{-rnd} = \{6\}$
$P_{-bck} = \{7\}$
$P_{-frnt} = \{8\}$

$$parent(x) = \begin{cases} 0 \Leftrightarrow x \in \{0,1,6,7,8\} \\ 1 \Leftrightarrow x = \{2, 3, 4\} \end{cases}$$



the following slides provide a transduction in quantifier-free first-order logic (QF) that translates between the unified model and the v-features model

the transduction: unified ⟶ v-features

$$
\begin{aligned}
\mathrm{rt}(x^1) &\stackrel{\text{def}}{=} \mathrm{rt}(x)\\
\mathrm{Place}(x^1) &\stackrel{\text{def}}{=} \text{C-place}(x)\\
+\mathrm{labial}(x^1) &\stackrel{\text{def}}{=} +\mathrm{labial}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
+\mathrm{coronal}(x^1) &\stackrel{\text{def}}{=} +\mathrm{coronal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
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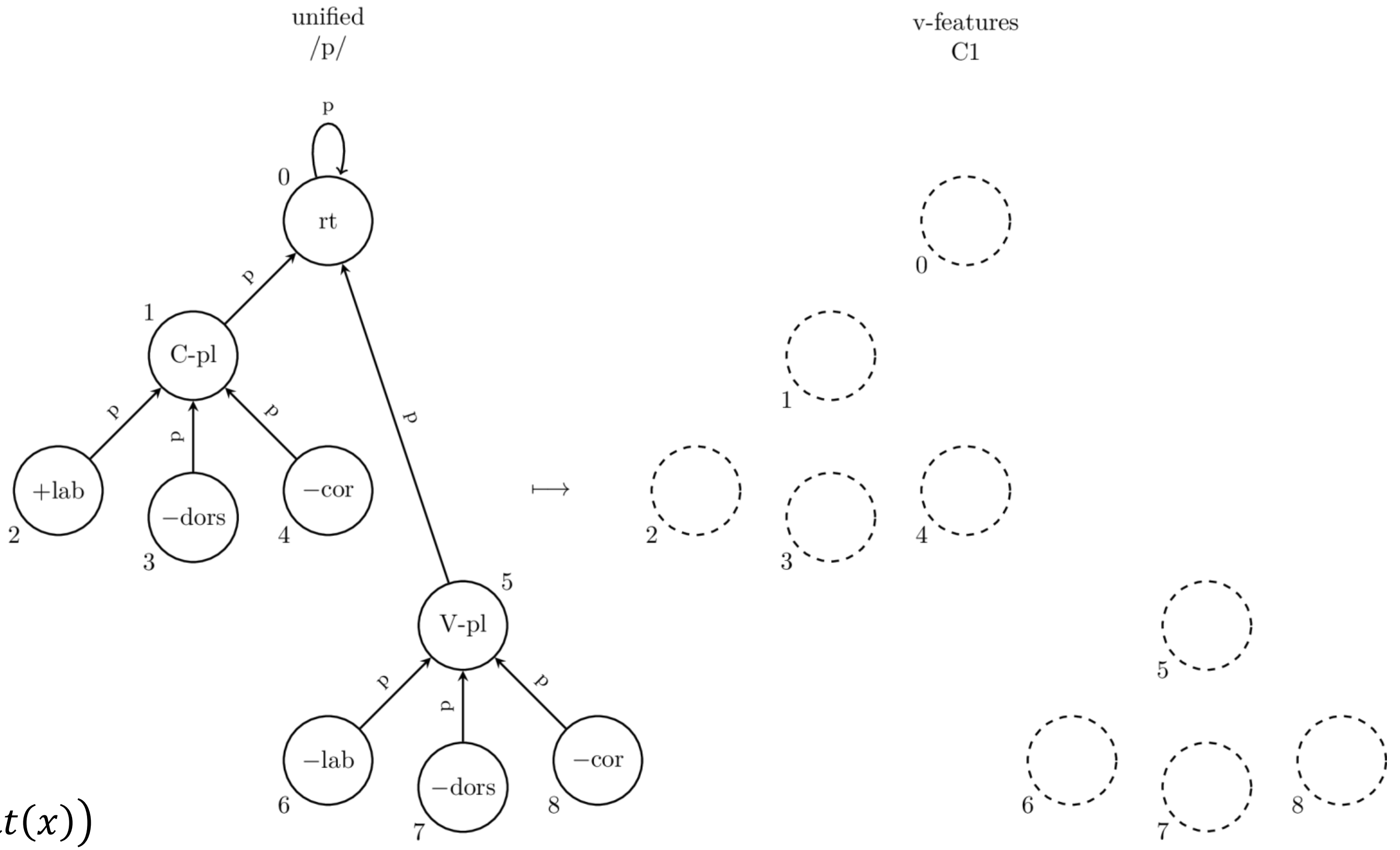

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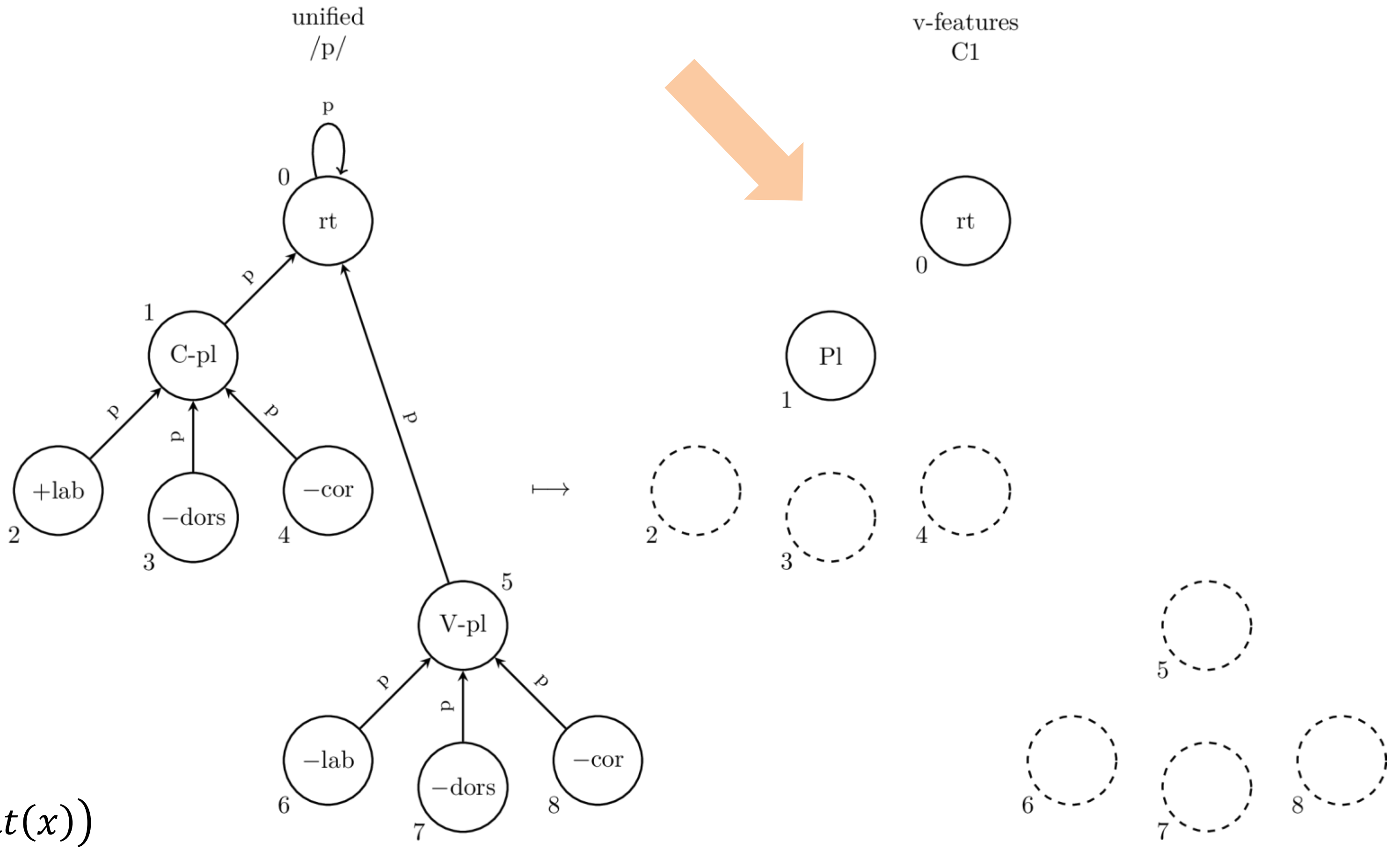

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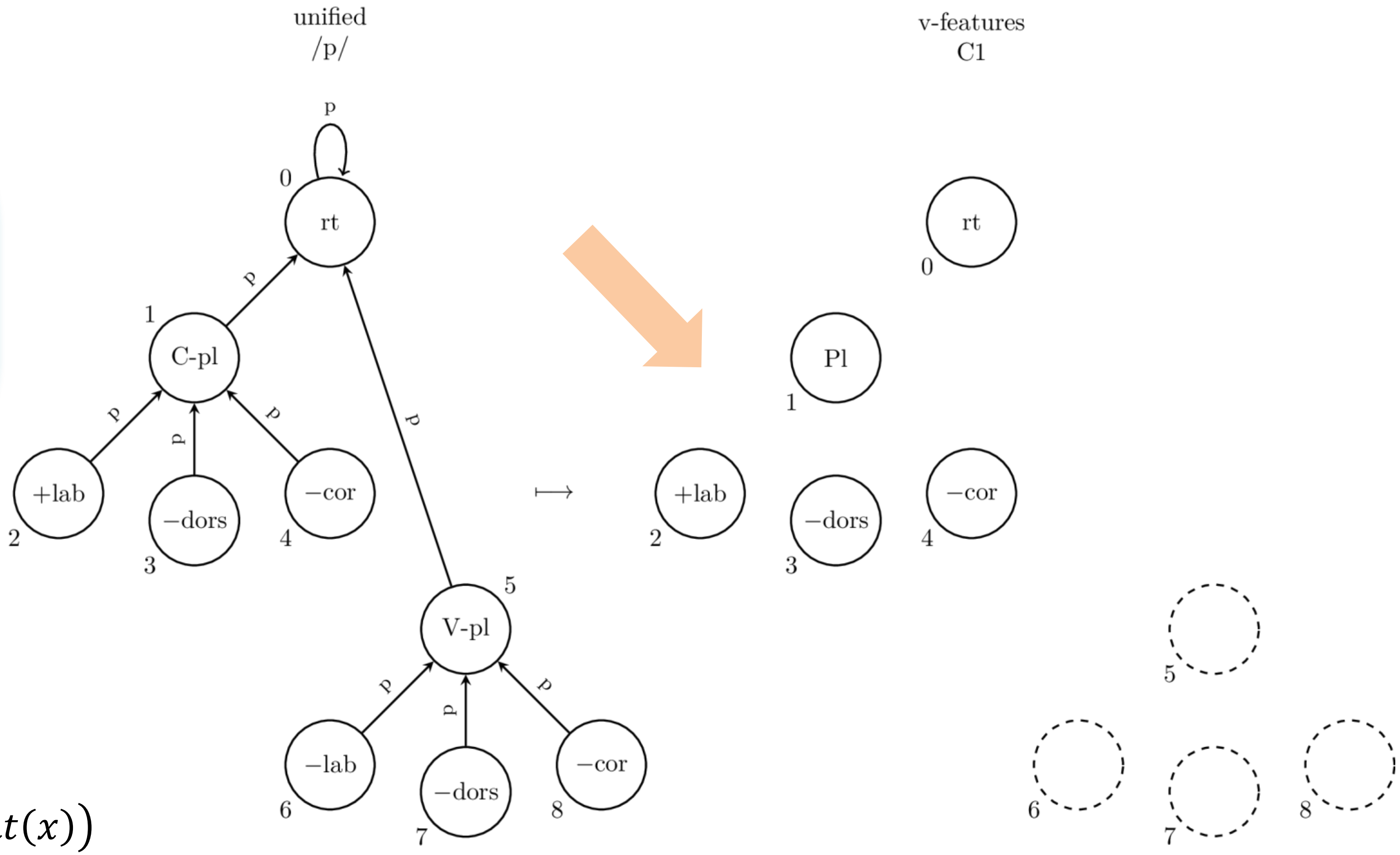

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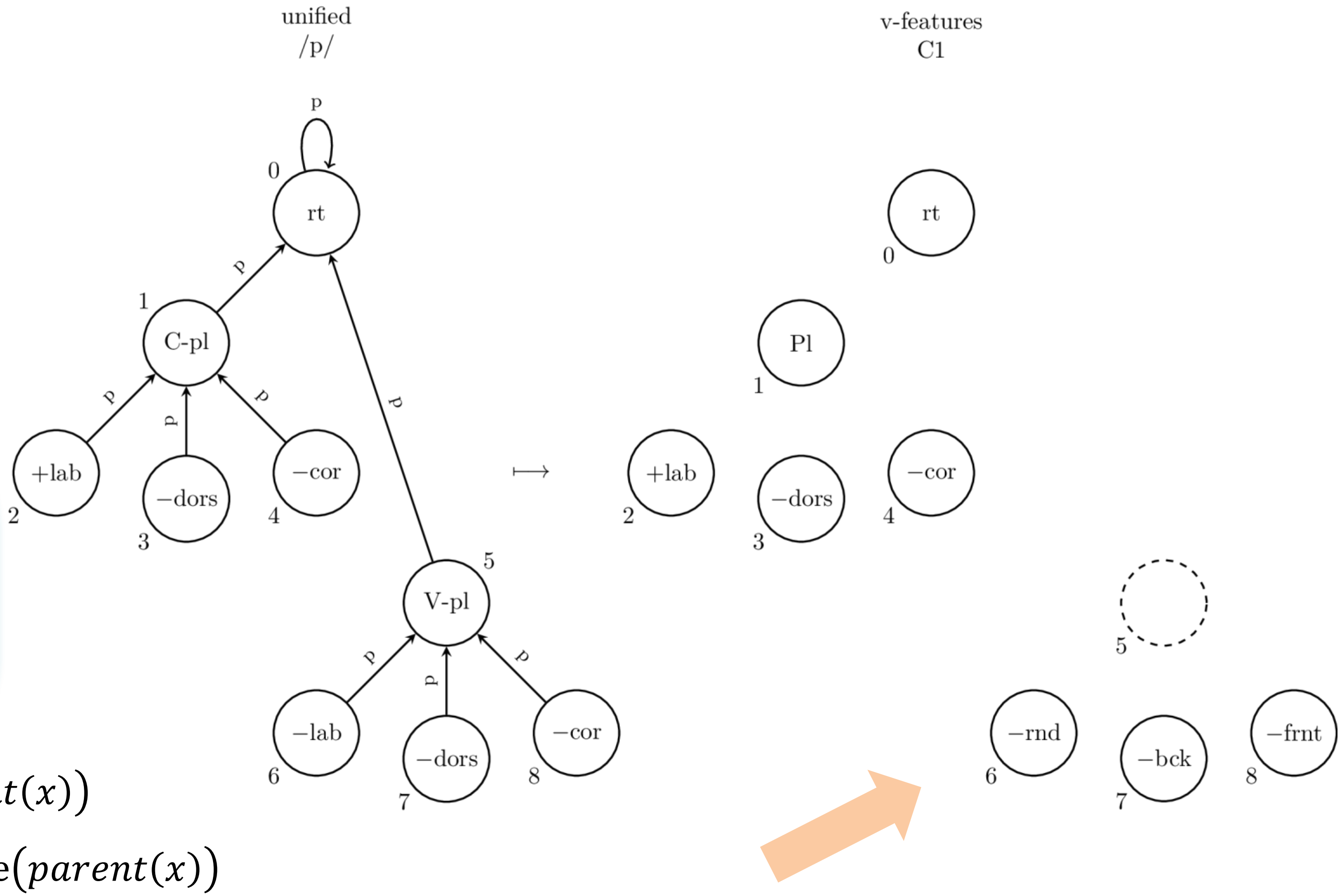

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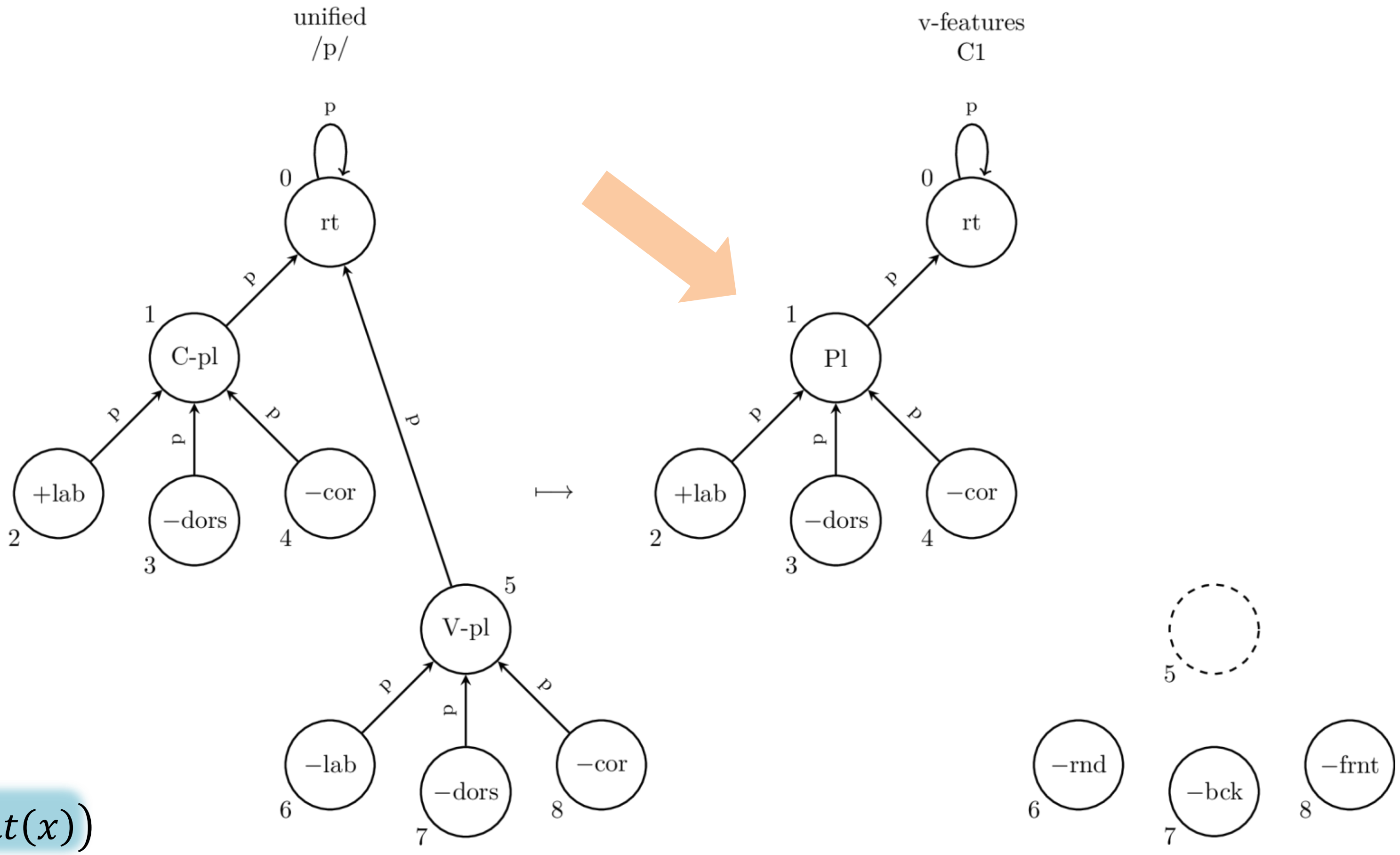

the transduction: unified ⟶ v-features

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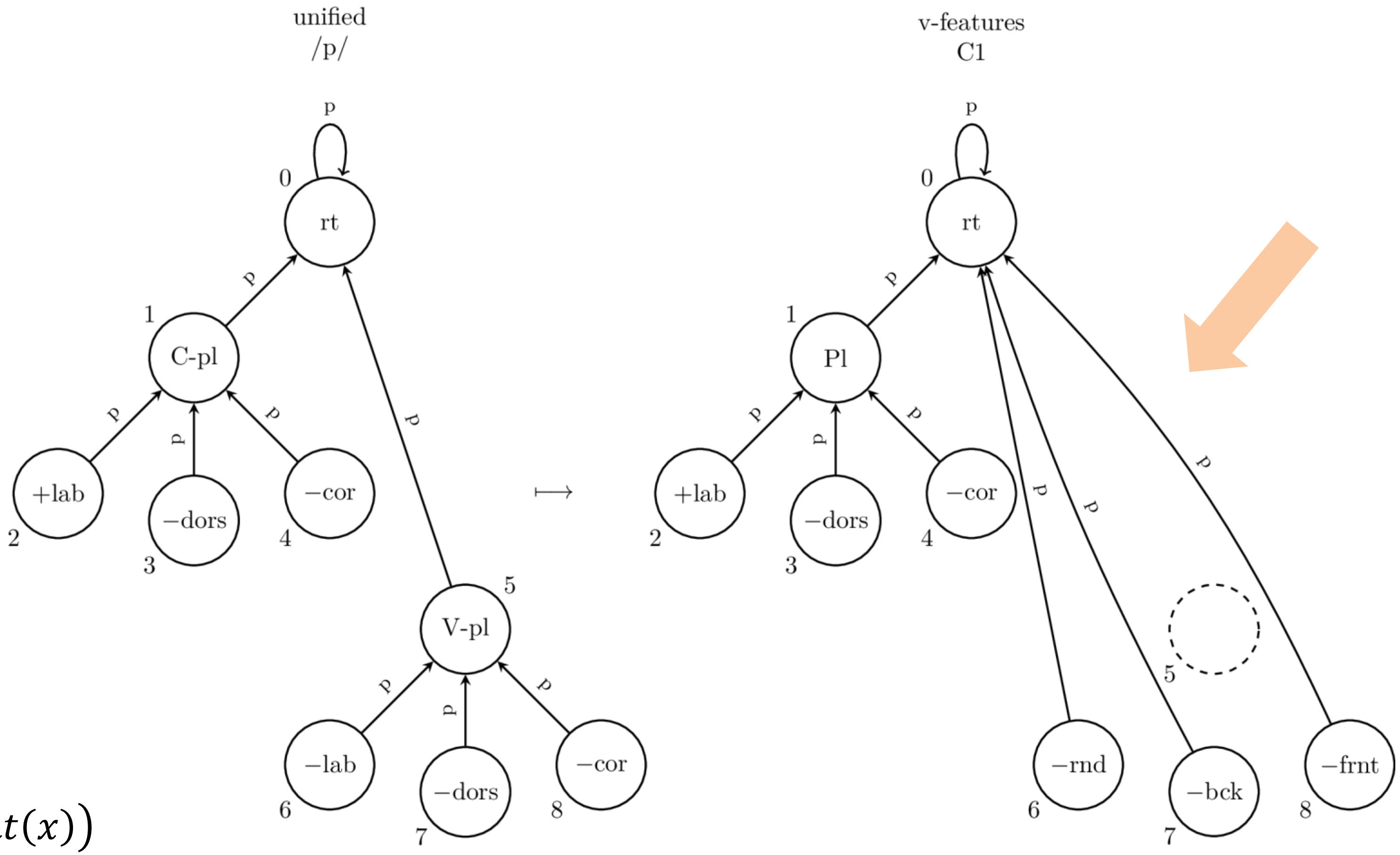

the transduction: unified → v-features

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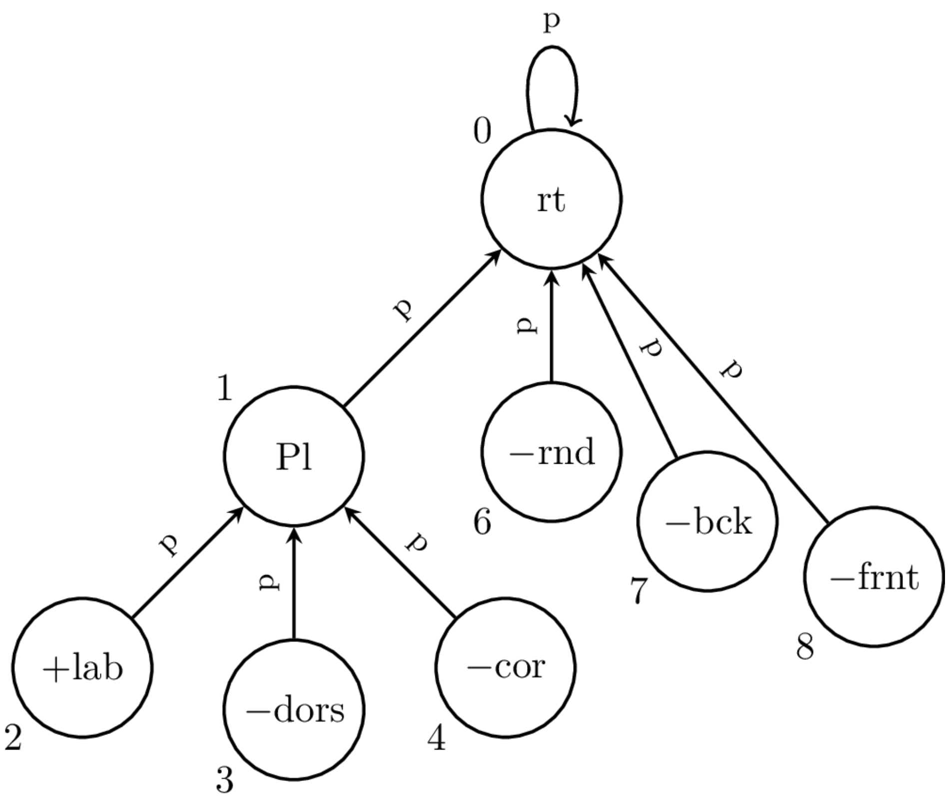
+\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{labial}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
+\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{coronal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
+\text{dorsal}(x^1) &\stackrel{\text{def}}{=} +\text{dorsal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
-\text{labial}(x^1) &\stackrel{\text{def}}{=} -\text{labial}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
-\text{coronal}(x^1) &\stackrel{\text{def}}{=} -\text{coronal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
-\text{dorsal}(x^1) &\stackrel{\text{def}}{=} -\text{dorsal}(x) \wedge \text{C-place}\big(\mathit{parent}(x)\big)\\
+\text{round}(x^1) &\stackrel{\text{def}}{=} +\text{labial}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
+\text{front}(x^1) &\stackrel{\text{def}}{=} +\text{coronal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
+\text{back}(x^1) &\stackrel{\text{def}}{=} +\text{dorsal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\text{round}(x^1) &\stackrel{\text{def}}{=} -\text{labial}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\text{front}(x^1) &\stackrel{\text{def}}{=} -\text{coronal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
-\text{back}(x^1) &\stackrel{\text{def}}{=} -\text{dorsal}(x) \wedge \text{V-place}\big(\mathit{parent}(x)\big)\\
\mathit{parent}(x^1) &\stackrel{\text{def}}{=} \begin{cases} \big(\mathit{parent}(x)\big)^1 \Leftrightarrow \neg\text{V-place}\big(\mathit{parent}(x)\big)\\ \Big(\mathit{parent}\big(\mathit{parent}(x)\big)\Big)^1 \Leftrightarrow \text{V-place}\big(\mathit{parent}(x)\big) \end{cases}
\end{aligned}
$$

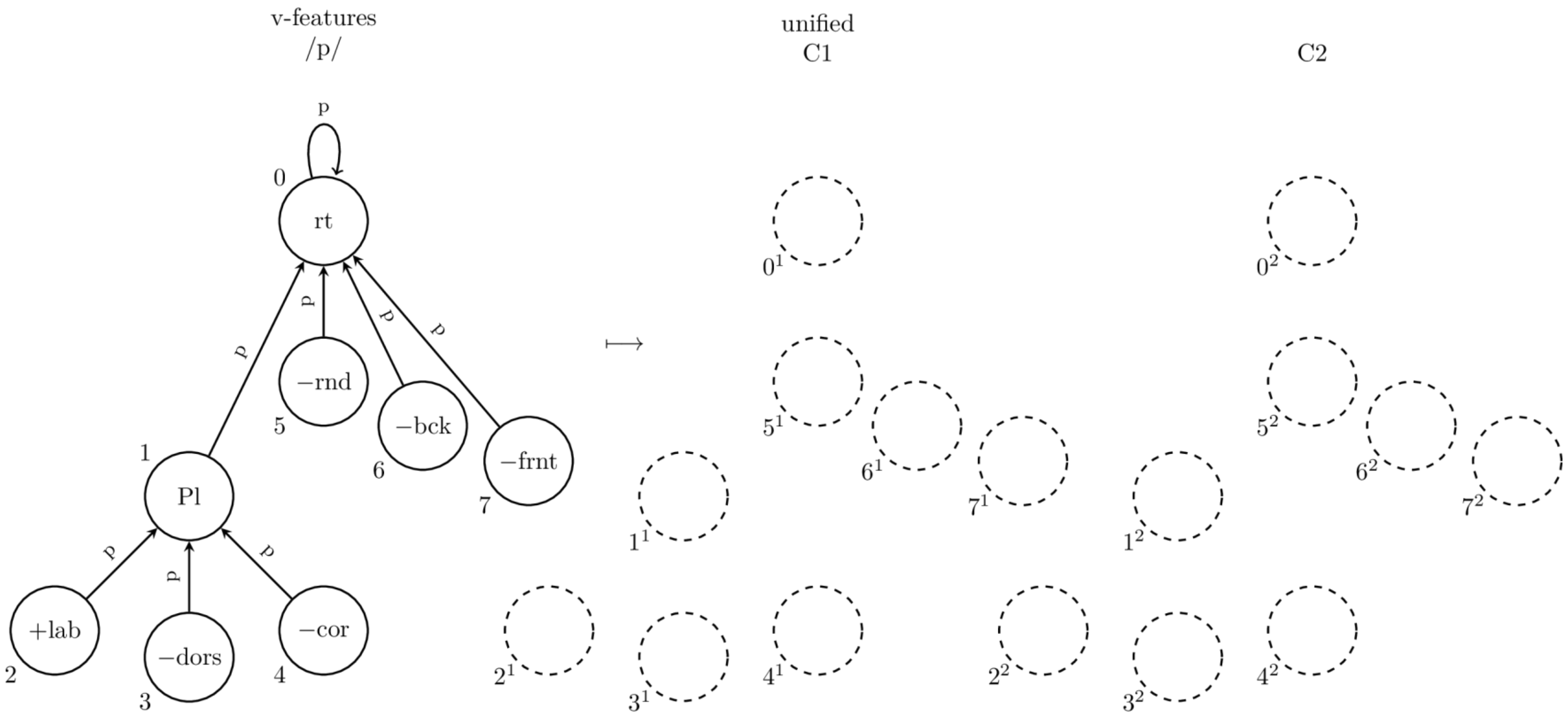
v-features
/p/



final output structure, rearranged with unlicensed nodes deleted

the transduction: v-features ⟶ unified

$$\mathrm{rt}(x^1) \stackrel{\text{def}}{=} \mathrm{rt}(x)$$
$$+\mathrm{labial}(x^1) \stackrel{\text{def}}{=} +\mathrm{round}(x) \lor \mathrm{labial}(x)$$
$$+\mathrm{coronal}(x^1) \stackrel{\text{def}}{=} +\mathrm{front}(x) \lor \mathrm{coronal}(x)$$
$$+\mathrm{dorsal}(x^1) \stackrel{\text{def}}{=} +\mathrm{back}(x) \lor \mathrm{coronal}(x)$$
$$-\mathrm{labial}(x^1) \stackrel{\text{def}}{=} -\mathrm{round}(x) \lor \mathrm{labial}(x)$$
$$-\mathrm{coronal}(x^1) \stackrel{\text{def}}{=} -\mathrm{front}(x) \lor \mathrm{coronal}(x)$$
$$-\mathrm{dorsal}(x^1) \stackrel{\text{def}}{=} -\mathrm{back}(x) \lor \mathrm{coronal}(x)$$
$$\text{C-place}(x^1) \stackrel{\text{def}}{=} \mathrm{Place}(x)$$
$$\text{V-place}(x^2) \stackrel{\text{def}}{=} \mathrm{rt}(x)$$
$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\mathsf{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathsf{vowelFeature}(x) \end{cases}$$
$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$



the transduction: v-features ⟶ unified

$$\mathrm{rt}(x^1) \overset{\text{def}}{=} \mathrm{rt}(x)$$

$$+\mathrm{labial}(x^1) \overset{\text{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)$$

$$+\mathrm{coronal}(x^1) \overset{\text{def}}{=} +\mathrm{front}(x) \vee \mathrm{coronal}(x)$$

$$+\mathrm{dorsal}(x^1) \overset{\text{def}}{=} +\mathrm{back}(x) \vee \mathrm{coronal}(x)$$

$$-\mathrm{labial}(x^1) \overset{\text{def}}{=} -\mathrm{round}(x) \vee \mathrm{labial}(x)$$

$$-\mathrm{coronal}(x^1) \overset{\text{def}}{=} -\mathrm{front}(x) \vee \mathrm{coronal}(x)$$

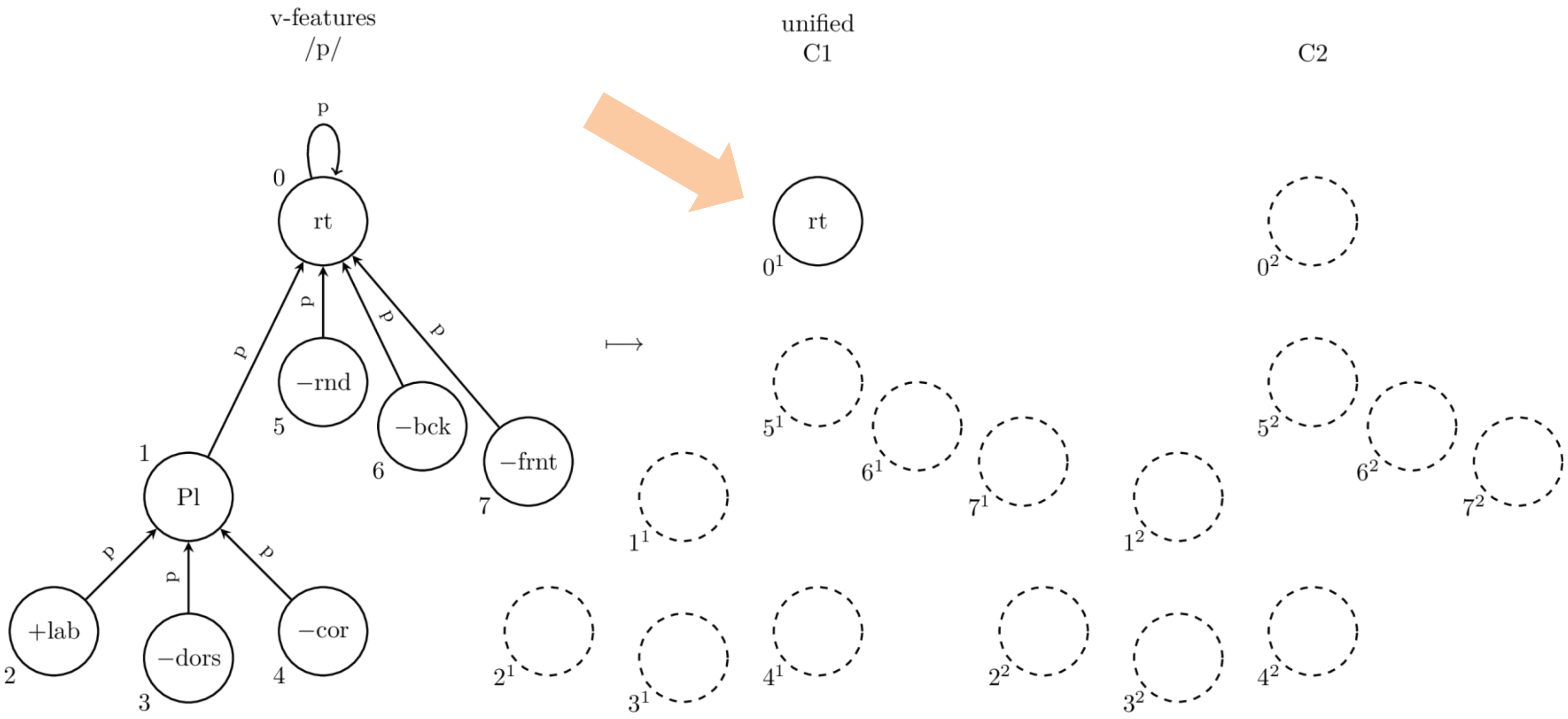
$$-\mathrm{dorsal}(x^1) \overset{\text{def}}{=} -\mathrm{back}(x) \vee \mathrm{coronal}(x)$$

$$\text{C-place}(x^1) \overset{\text{def}}{=} \mathrm{Place}(x)$$

$$\text{V-place}(x^2) \overset{\text{def}}{=} \mathrm{rt}(x)$$

$$parent(x^1) \overset{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\mathrm{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \overset{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$



the transduction: v-features ⟶ unified

$$\mathrm{rt}(x^1) \overset{\mathrm{def}}{=} \mathrm{rt}(x)$$

$$+\mathrm{labial}(x^1) \overset{\mathrm{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)$$

$$+\mathrm{coronal}(x^1) \overset{\mathrm{def}}{=} +\mathrm{front}(x) \vee \mathrm{coronal}(x)$$

$$+\mathrm{dorsal}(x^1) \overset{\mathrm{def}}{=} +\mathrm{back}(x) \vee \mathrm{coronal}(x)$$

$$-\mathrm{labial}(x^1) \overset{\mathrm{def}}{=} -\mathrm{round}(x) \vee \mathrm{labial}(x)$$

$$-\mathrm{coronal}(x^1) \overset{\mathrm{def}}{=} -\mathrm{front}(x) \vee \mathrm{coronal}(x)$$

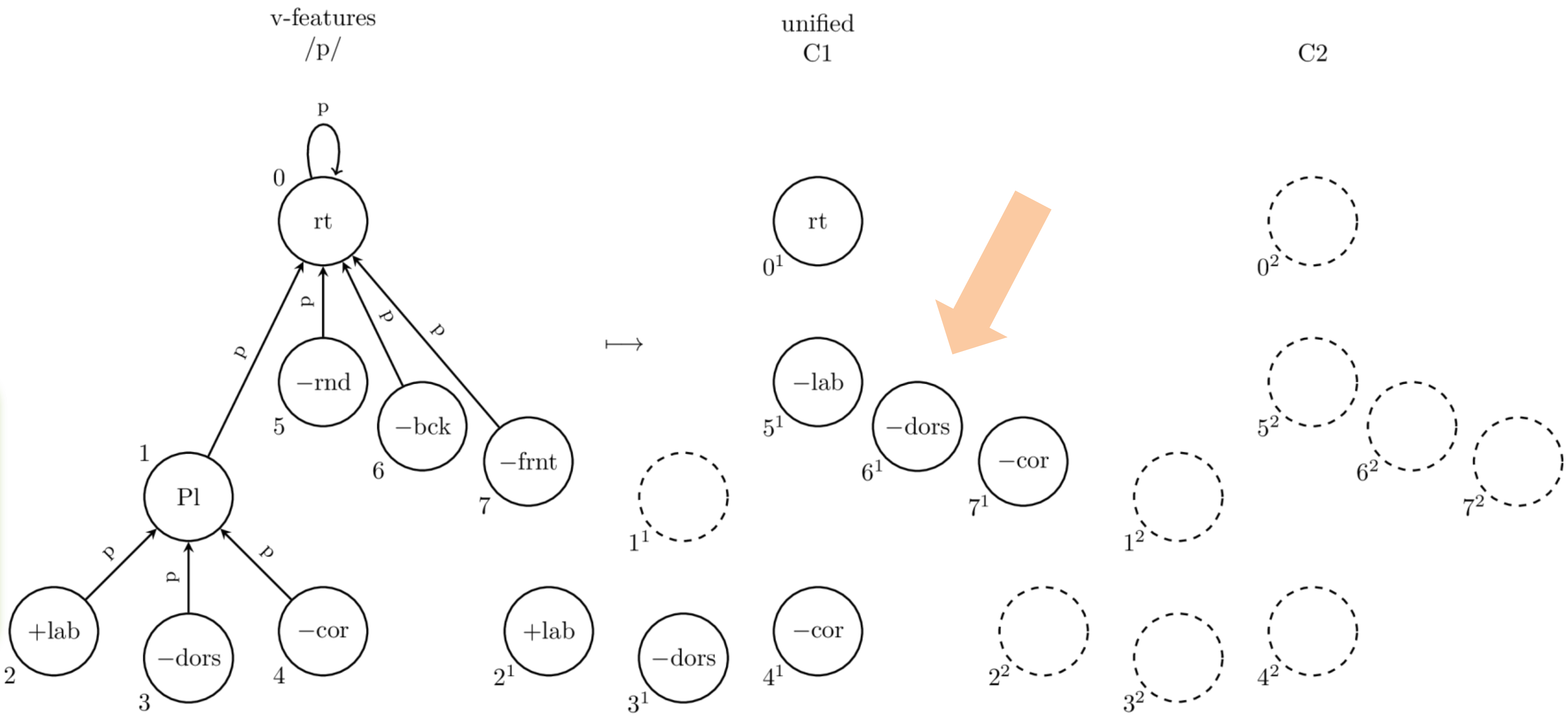
$$-\mathrm{dorsal}(x^1) \overset{\mathrm{def}}{=} -\mathrm{back}(x) \vee \mathrm{coronal}(x)$$

$$\text{C-place}(x^1) \overset{\mathrm{def}}{=} \mathrm{Place}(x)$$

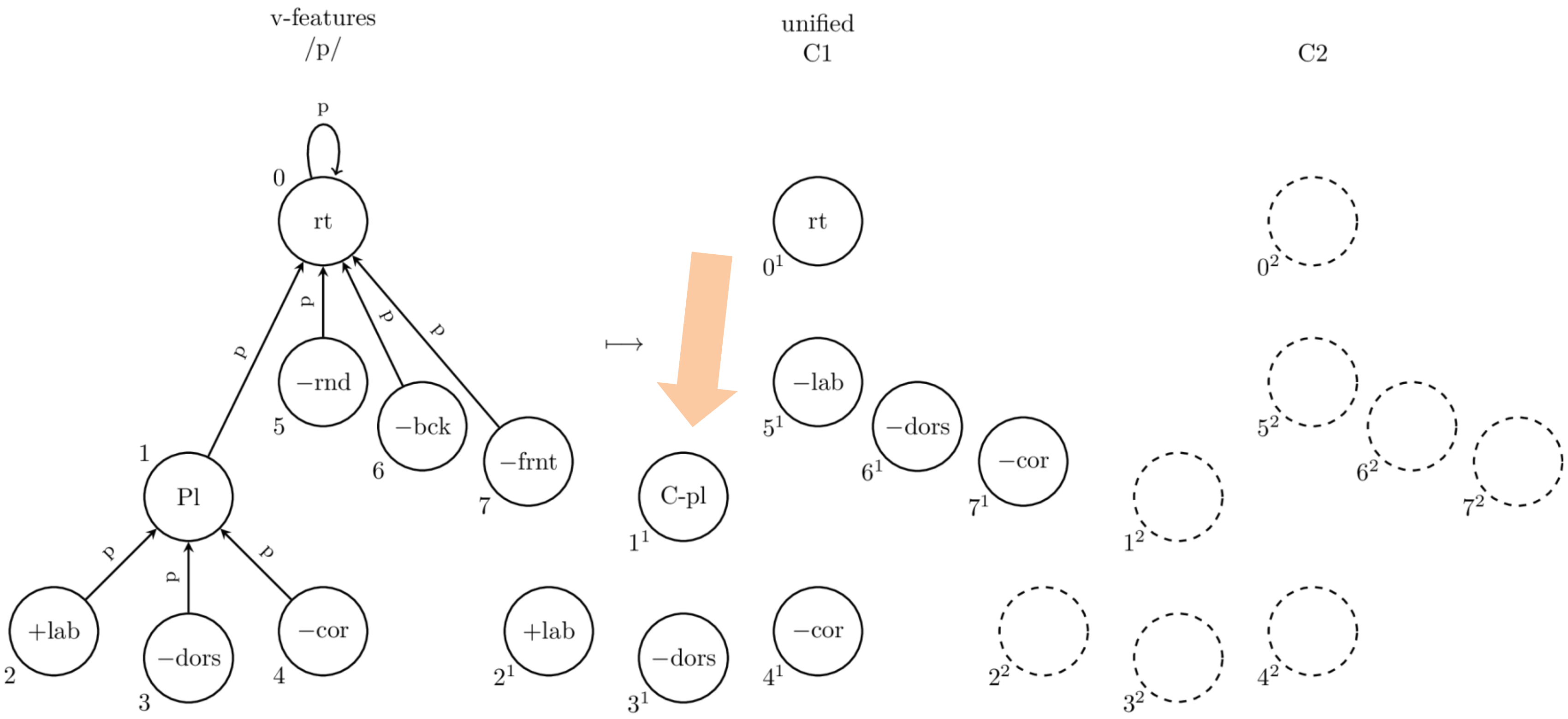
$$\text{V-place}(x^2) \overset{\mathrm{def}}{=} \mathrm{rt}(x)$$

$$parent(x^1) \overset{\mathrm{def}}{=} \begin{cases} \big(parent(x)\big)^1 \Leftrightarrow \neg\mathrm{vowelFeature}(x)) \\ \big(parent(x)\big)^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \overset{\mathrm{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

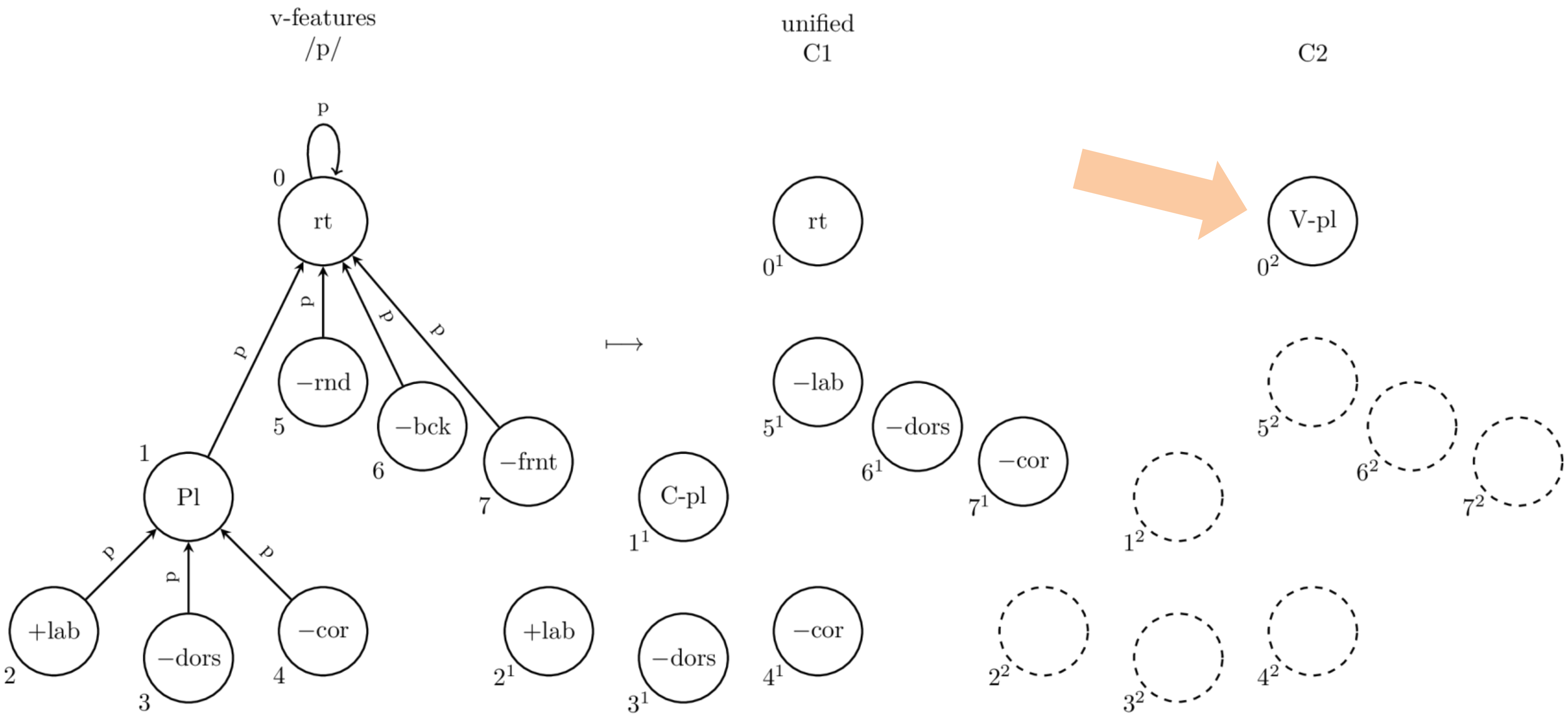


the transduction: v-features ⟶ unified



$$\text{rt}(x^1) \stackrel{\text{def}}{=} \text{rt}(x)$$
$$+\text{labial}(x^1) \stackrel{\text{def}}{=} +\text{round}(x) \lor \text{labial}(x)$$
$$+\text{coronal}(x^1) \stackrel{\text{def}}{=} +\text{front}(x) \lor \text{coronal}(x)$$
$$+\text{dorsal}(x^1) \stackrel{\text{def}}{=} +\text{back}(x) \lor \text{coronal}(x)$$
$$-\text{labial}(x^1) \stackrel{\text{def}}{=} -\text{round}(x) \lor \text{labial}(x)$$
$$-\text{coronal}(x^1) \stackrel{\text{def}}{=} -\text{front}(x) \lor \text{coronal}(x)$$
$$-\text{dorsal}(x^1) \stackrel{\text{def}}{=} -\text{back}(x) \lor \text{coronal}(x)$$
$$\text{C-place}(x^1) \stackrel{\text{def}}{=} \text{Place}(x)$$
$$\text{V-place}(x^2) \stackrel{\text{def}}{=} \text{rt}(x)$$
$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\text{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases}$$
$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)$$

the transduction: v-features ⟶ unified



$$\text{rt}(x^1) \stackrel{\text{def}}{=} \text{rt}(x)$$

$$+\text{labial}(x^1) \stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x)$$

$$+\text{coronal}(x^1) \stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x)$$

$$+\text{dorsal}(x^1) \stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x)$$

$$-\text{labial}(x^1) \stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x)$$

$$-\text{coronal}(x^1) \stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x)$$

$$-\text{dorsal}(x^1) \stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x)$$

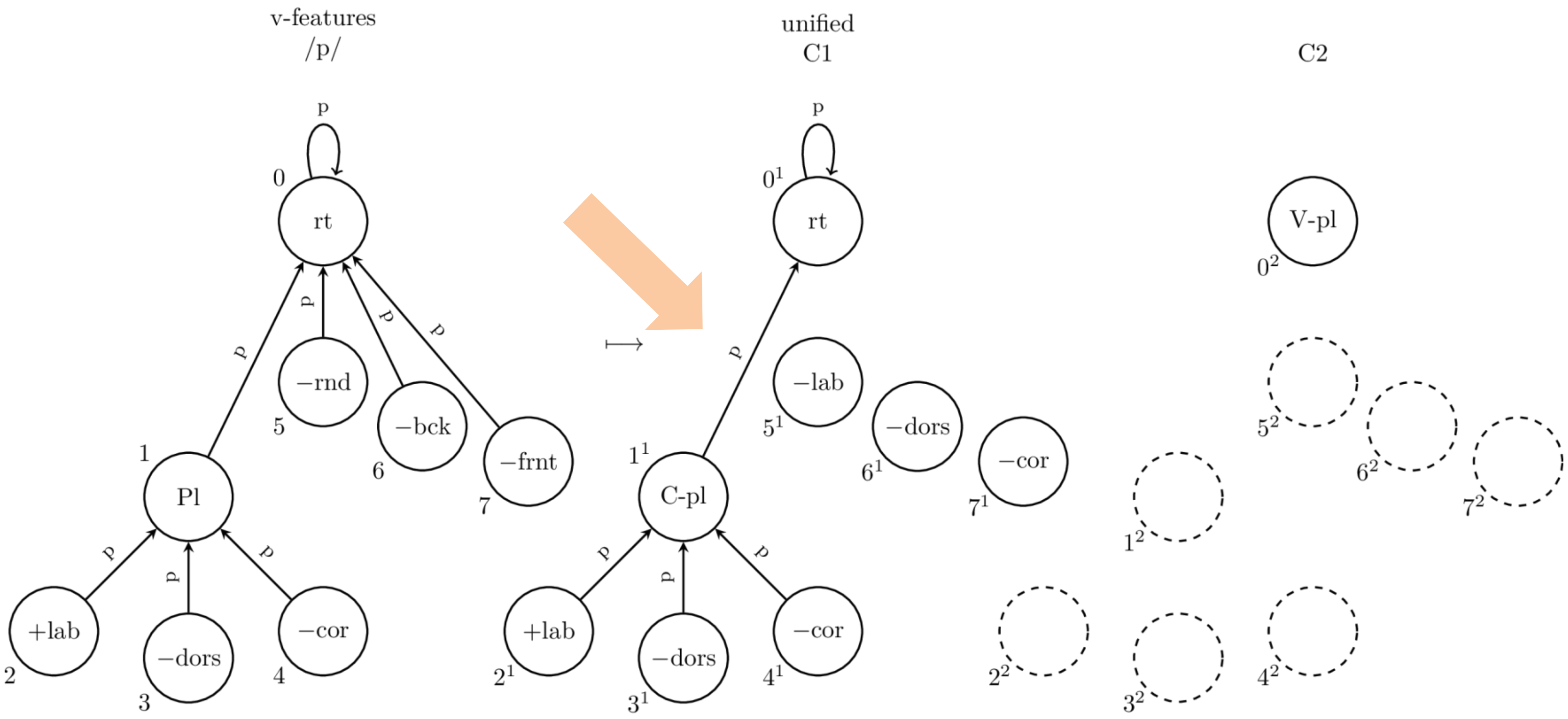
$$\text{C-place}(x^1) \stackrel{\text{def}}{=} \text{Place}(x)$$

$$\text{V-place}(x^2) \stackrel{\text{def}}{=} \text{rt}(x)$$

$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\text{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)$$

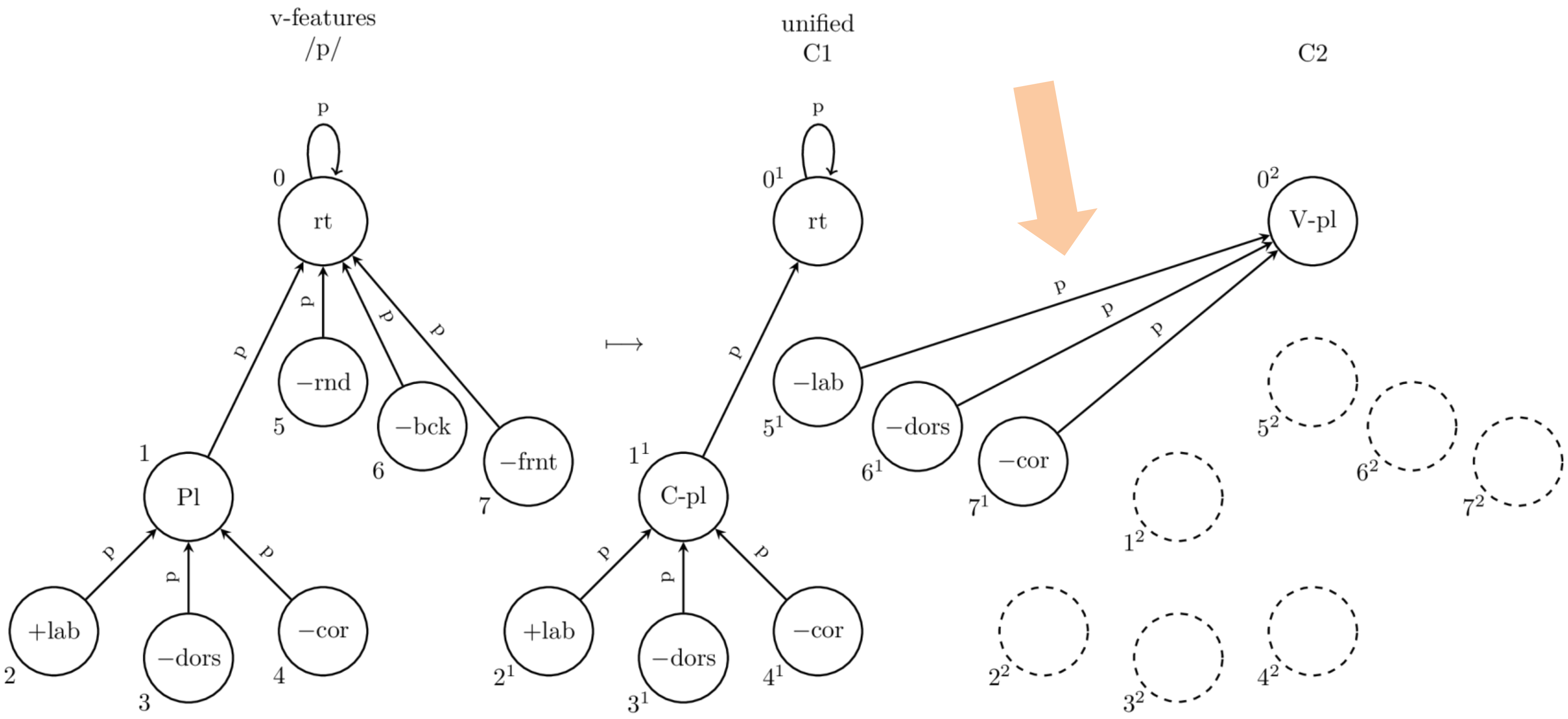
the transduction: v-features ⟶ unified



$$
\begin{aligned}
\mathrm{rt}(x^1) &\overset{\text{def}}{=} \mathrm{rt}(x)\\
+\mathrm{labial}(x^1) &\overset{\text{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)\\
+\mathrm{coronal}(x^1) &\overset{\text{def}}{=} +\mathrm{front}(x) \vee \mathrm{coronal}(x)\\
+\mathrm{dorsal}(x^1) &\overset{\text{def}}{=} +\mathrm{back}(x) \vee \mathrm{coronal}(x)\\
-\mathrm{labial}(x^1) &\overset{\text{def}}{=} -\mathrm{round}(x) \vee \mathrm{labial}(x)\\
-\mathrm{coronal}(x^1) &\overset{\text{def}}{=} -\mathrm{front}(x) \vee \mathrm{coronal}(x)\\
-\mathrm{dorsal}(x^1) &\overset{\text{def}}{=} -\mathrm{back}(x) \vee \mathrm{coronal}(x)\\
\text{C-place}(x^1) &\overset{\text{def}}{=} \mathrm{Place}(x)\\
\text{V-place}(x^2) &\overset{\text{def}}{=} \mathrm{rt}(x)\\
parent(x^1) &\overset{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg \mathrm{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}\\
parent(x^2) &\overset{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)
\end{aligned}
$$

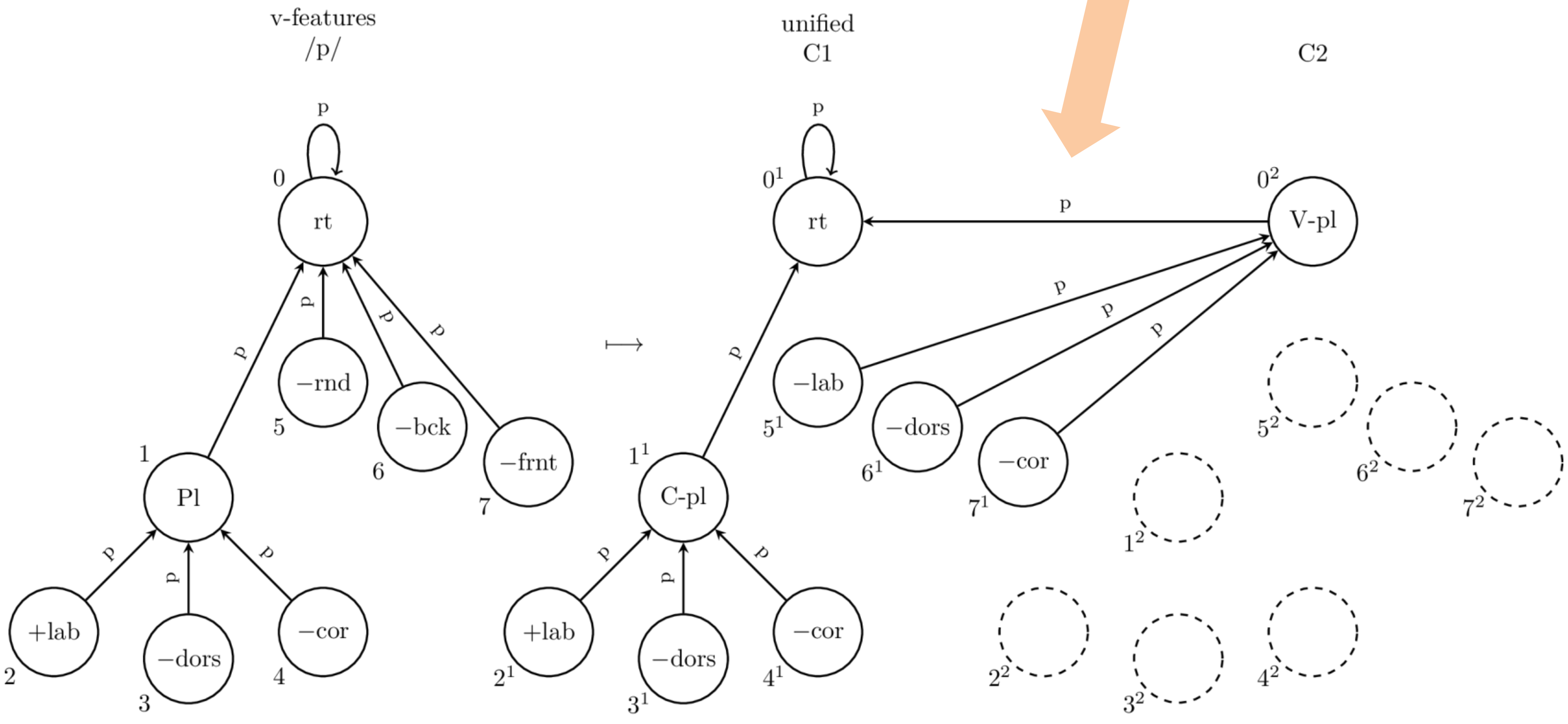
$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \vee -\mathrm{round}(x) \vee +\mathrm{front}(x) \vee -\mathrm{front}(x) \vee +\mathrm{back}(x) \vee -\mathrm{back}(x)$$

the transduction: v-features ⟶ unified



$$\mathrm{rt}(x^1) \stackrel{\mathrm{def}}{=} \mathrm{rt}(x)$$
$$+\mathrm{labial}(x^1) \stackrel{\mathrm{def}}{=} +\mathrm{round}(x) \lor \mathrm{labial}(x)$$
$$+\mathrm{coronal}(x^1) \stackrel{\mathrm{def}}{=} +\mathrm{front}(x) \lor \mathrm{coronal}(x)$$
$$+\mathrm{dorsal}(x^1) \stackrel{\mathrm{def}}{=} +\mathrm{back}(x) \lor \mathrm{coronal}(x)$$
$$-\mathrm{labial}(x^1) \stackrel{\mathrm{def}}{=} -\mathrm{round}(x) \lor \mathrm{labial}(x)$$
$$-\mathrm{coronal}(x^1) \stackrel{\mathrm{def}}{=} -\mathrm{front}(x) \lor \mathrm{coronal}(x)$$
$$-\mathrm{dorsal}(x^1) \stackrel{\mathrm{def}}{=} -\mathrm{back}(x) \lor \mathrm{coronal}(x)$$
$$\text{C-place}(x^1) \stackrel{\mathrm{def}}{=} \mathrm{Place}(x)$$
$$\text{V-place}(x^2) \stackrel{\mathrm{def}}{=} \mathrm{rt}(x)$$
$$parent(x^1) \stackrel{\mathrm{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\mathrm{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}$$
$$parent(x^2) \stackrel{\mathrm{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$
$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \lor -\mathrm{round}(x) \lor +\mathrm{front}(x) \lor -\mathrm{front}(x) \lor +\mathrm{back}(x) \lor -\mathrm{back}(x)$$

the transduction: v-features ⟶ unified



$$\mathrm{rt}(x^1) \stackrel{\text{def}}{=} \mathrm{rt}(x)$$
$$+\mathrm{labial}(x^1) \stackrel{\text{def}}{=} +\mathrm{round}(x) \vee \mathrm{labial}(x)$$
$$+\mathrm{coronal}(x^1) \stackrel{\text{def}}{=} +\mathrm{front}(x) \vee \mathrm{coronal}(x)$$
$$+\mathrm{dorsal}(x^1) \stackrel{\text{def}}{=} +\mathrm{back}(x) \vee \mathrm{coronal}(x)$$
$$-\mathrm{labial}(x^1) \stackrel{\text{def}}{=} -\mathrm{round}(x) \vee \mathrm{labial}(x)$$
$$-\mathrm{coronal}(x^1) \stackrel{\text{def}}{=} -\mathrm{front}(x) \vee \mathrm{coronal}(x)$$
$$-\mathrm{dorsal}(x^1) \stackrel{\text{def}}{=} -\mathrm{back}(x) \vee \mathrm{coronal}(x)$$
$$\text{C-place}(x^1) \stackrel{\text{def}}{=} \mathrm{Place}(x)$$
$$\text{V-place}(x^2) \stackrel{\text{def}}{=} \mathrm{rt}(x)$$
$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 \Leftrightarrow \neg\mathrm{vowelFeature}(x)) \\ (parent(x))^2 \Leftrightarrow \mathrm{vowelFeature}(x) \end{cases}$$
$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \mathrm{rt}(x)$$

$$\mathrm{vowelFeature}(x) = +\mathrm{round}(x) \vee -\mathrm{round}(x) \vee +\mathrm{front}(x) \vee -\mathrm{front}(x) \vee +\mathrm{back}(x) \vee -\mathrm{back}(x)$$