Comparing phonological representations

Nick Danis nsdanis@wustl.edu

Graduate Linguistic Expo at Michigan State (GLEAMS) October 28-29, 2022

Washington University in St. Louis





consonants and vowels probably should be natural classes



consonants and vowels probably *shouldn't* be natural classes





big questions

- how can we formally compare phonological representations?
- what can we learn from these comparisons?
- what do we care about as linguists?
- why care about anything?



medium answers

- two theories can be shown to be formally equivalent using logic and model theory
 - given two representations A and B, a transduction between A and B means that any linguistic rule given with structure A can be translated into structure B, and vice versa
 - Strother-Garcia (2019), Danis & Jardine (2019), Oakden (2020), a.o.





(Strother-Garcia 2019)

medium answers

- not every transduction preserves ideas of *linguistic* equivalence
 - process should respect natural classes, which may be lost in certain transductions
- the property of a natural-class preserving transduction is defined to find those logically equivalent representations that also share linguistic intuitions





introduction foundations

example transduction existing transductions broader significance conclusion

- segments have structure
- some segments potentially share structure
- a natural class is a set of all segments that share some piece of structure



natural-class preserving transductions



- A transduction between two representational theories A and B is **naturalclass preserving** iff the set of all natural class extensions of A exactly match those of B
 - A natural class extension is an exhaustive set of atomic segments that map to feature structures that share some common structural property

		[+front]	[-front]	[-front]
[-low]	[+high]	i		u
[-low]	[-high]	e		Ο
[+low]	[-high]		a	

shared structur	e: [+high]		
natural class: {	[−low] +high +front]	,	[−low] +high −front_	}
natural class extension: { i u }				

"In view of this, if a theory of language failed to provide a mechanism for making distinctions between more or less natural classes of segments, this failure would be sufficient reason for rejecting the theory as being incapable of attaining the level of explanatory adequacy."

(Chomsky & Halle 1968: 355)









"This combinability of features allows phonology to construct complex symbols from an inventory of simple parts, and provides an explanation for the so-called natural class behavior—**different structures can behave alike because they contain identical substructures**."

" In Logical Phonology (see section 3), rules refer to natural classes by definition: a statement that cannot be formulated in terms of natural classes is not a rule."

(Volenec & Reiss 2020: 22, 28)

"...that consequently the whole history of mankind [...] has been a history of class struggles."

(Marx 1848:8)







natural classes as a computational learning bias

"Without an ability to use knowledge about phonological features to generalize across phones, OSTIA's transducers have missing transitions for certain phones from certain states. This causes errors when transducing previously unseen words after training is complete." (Gildea & Jurafsky 1996)





natural classes as a computational learning bias



transducer learned with no natural class knowledge

transducer learned with knowledge of natural classes

assimilation: act naturally

- assimilation operates over like things
 - Trubetzkoy (1969), Chomsky and Halle (1968), Hyman (1974), Hayes (1986), Clements & Hume (1995), me right now, a.o.



Α

F



or

F

Sharing is Caring the structural changes on the target of an assimilation

process should be factors of the trigger

assimilation: sharing is caring



- Clements & Hume (1995):
 - "Phonological rules perform single operations only." (p. 250)
 - "In the present model, in contrast, assimilation rules are characterized as the association (or "spreading") of a feature or node F of segment A to a neighboring segment B..." (p. 258)
- If assimilation is the result of spreading (the addition of an association relation), then it directly follows from this that the resulting segments will have shared structure and therefore constitute a nontrivial natural class

the general argument



- 1. if we assume a nontrivial theory of segmental structure, and
- 2. if we assume for assimilation that sharing is caring 🔊
- 3. then the range of possible assimilation processes is restricted

further:

- 4. if two theories are shown to be logically equivalent, and
- 5. if this transduction is not natural-class preserving
- 6. then the two theories do not make the same empirical predictions (by 3)

introduction foundations **example transduction** existing transductions broader significance conclusion

comparing theories



unified place theory

- consonants and vowels share representational primitives
 - e.g. LABIAL C-place, LABIAL V-place
- Sagey (1986), Clements & Hume (1995), a.o.

vowel features theory

- vowel place is largely defined by primitives not used to describe consonant place
 a g [uback] [-round]
 - e.g. [+back], [-round]
- Odden (1991), Ni Chiosain & Padgett (1993), Halle et al. (2000), a.o.



crucial difference: **unified** uses same features for vocalic and consonantal contrasts

orthogonal issues:

- binary vs. privative features
- underspecification

comparing theories

- each theory is translated into a **finite model** defining the domain of nodes, relations, and functions in each
- each model defines a logical language for each theory of representation
- a **transduction** translates all relations & functions in one model to the other
- any sentence/rule/constraint expressible in one model is therefore expressible in the other



comparing theories

$$D = \{0, 1, 2, 3, 4, 6, 7\}$$

$$P_{rt} = \{0\}$$

$$P_{Pl} = \{1\}$$

$$P_{+lab} = \{2\}$$

$$P_{-dors} = \{3\}$$

$$P_{-cor} = \{4\}$$

$$P_{-rnd} = \{6\}$$

$$P_{-bck} = \{7\}$$

$$P_{-frnt} = \{8\}$$

$$parent(x) = \begin{cases} 0 \Leftrightarrow x \in \{0, 1, 6, 7, 8\} \\ 1 \Leftrightarrow x = \{2, 3, 4\} \end{cases}$$





the following slides provide a transduction in quantifier-free first-order logic (QF) that translates between the unified model and the v-features model



 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ +round(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land V-place(parent(x)) +lab+front(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land V-place(parent(x)) +back $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge V - \operatorname{place}(parent(x))$ $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \wedge \text{V-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-place}(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-place}(parent(x)) \end{cases}$







 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \wedge C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ +round(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land V-place(parent(x)) +front $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{coronal}(x) \wedge \operatorname{V-place}(parent(x))$ +back $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge V - \operatorname{place}(parent(x))$ $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \land V\text{-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-}place(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-}place(parent(x)) \end{cases}$





 $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \wedge C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) +dorsal(x^1) $\stackrel{\text{def}}{=}$ +corsal(x) \land C-place(parent(x)) $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ $-coronal(x^1) \stackrel{\text{\tiny def}}{=} -coronal(x) \wedge C-place(parent(x))$ $-dorsal(x^1) \stackrel{\text{\tiny def}}{=} -dorsal(x) \wedge C-place(parent(x))$ +round(x^1) $\stackrel{\text{def}}{=}$ +labial(x) \land V-place(parent(x)) +front $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{coronal}(x) \wedge \operatorname{V-place}(parent(x))$ +back $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge V - \operatorname{place}(parent(x))$ $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \land V\text{-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-}place(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-}place(parent(x)) \end{cases}$

 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$



-frnt

the transduction: unified \rightarrow v-features

unified v-features /p/ C1 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ C-pl \mathbf{Pl} $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ d +round(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land V-place(parent(x)) +lab+lab-cor-dors -dors +front(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land V-place(parent(x)) +back(x^1) $\stackrel{\text{def}}{=}$ +corsal(x) \land V-place(parent(x)) V-pl $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \land \text{V-place}(parent(x))$ d $-back(x^1) \stackrel{\text{\tiny def}}{=} -dorsal(x) \wedge V-place(parent(x))$ -lab -cor $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-place}(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-place}(parent(x)) \end{cases}$



-frnt

the transduction: unified \rightarrow v-features

unified v-features /p/ C1 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ C-pl \mathbf{Pl} $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ d d +round(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land V-place(parent(x)) +lab+lab-cor-cor-dors -dors +front $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{coronal}(x) \wedge \operatorname{V-place}(parent(x))$ +back $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge V - \operatorname{place}(parent(x))$ V-pl $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \land V\text{-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ -lab -cor $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-place}(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-place}(parent(x)) \end{cases}$



 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land C-place(parent(x)) +coronal(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ C-pl $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ d +round(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land V-place(parent(x)) +lab -dors +front(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land V-place(parent(x)) +back $(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge V - \operatorname{place}(parent(x))$ $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \wedge \text{V-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-}place(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-}place(parent(x)) \end{cases}$





 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ $Place(x^1) \stackrel{\text{\tiny def}}{=} C-place(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +labial(x) \land C-place(parent(x)) +coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +coronal(x) \land C-place(parent(x)) $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{corsal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{labial}(x) \wedge \text{C-place}(parent(x))$ $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{coronal}(x) \wedge \operatorname{C-place}(parent(x))$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{dorsal}(x) \wedge \operatorname{C-place}(parent(x))$ +round(x^1) $\stackrel{\text{def}}{=}$ +labial(x) \land V-place(parent(x)) +front(x^1) $\stackrel{\text{def}}{=}$ +coronal(x) \land V-place(parent(x)) +back(x^1) $\stackrel{\text{def}}{=}$ +corsal(x) \land V-place(parent(x)) $-\operatorname{round}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{labial}(x) \wedge \operatorname{V-place}(parent(x))$ $-\text{front}(x^1) \stackrel{\text{\tiny def}}{=} -\text{coronal}(x) \land V\text{-place}(parent(x))$ $-\text{back}(x^1) \stackrel{\text{\tiny def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(parent(x))$ $parent(x^{1}) \stackrel{\text{\tiny def}}{=} \begin{cases} \left(parent(x)\right)^{1} \Leftrightarrow \neg V\text{-}place(parent(x)) \\ \left(parent(parent(x))\right)^{1} \Leftrightarrow V\text{-}place(parent(x)) \end{cases}$



v-features

final output structure, rearranged with unlicensed nodes deleted

the transduction: v-features \rightarrow unified



17 II

 $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

17 II the transduction: v-features \rightarrow unified v-features unified /p/ C2C1 \mathbf{rt} $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +round(x) \lor labial(x) -rnd -bck +coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +front(x) \lor coronal(x) -frnt $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{back}(x) \lor \operatorname{coronal}(x)$ Pl $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{round}(x) \lor \text{labial}(x)$ d $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{front}(x) \lor \operatorname{coronal}(x)$ 2^1 2^2 2^2 2^2 2^2 2^2 +lab $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{back}(x) \lor \operatorname{coronal}(x)$ -cor-dorsC-place(x^1) $\stackrel{\text{\tiny def}}{=}$ Place(x) V-place(x^2) $\stackrel{\text{\tiny def}}{=}$ rt(x) $parent(x^{1}) \stackrel{\text{\tiny def}}{=} \begin{cases} \left(parent(x) \right)^{1} \Leftrightarrow \neg \text{vowelFeature}(x) \\ \left(parent(x) \right)^{2} \Leftrightarrow \text{vowelFeature}(x) \end{cases}$ $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

☆ \\ \\ \\ the transduction: v-features \rightarrow unified v-features unified /p/ C2C1 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ +labial(x^1) $\stackrel{\text{def}}{=}$ +round(x) \lor labial(x) $-\mathrm{rnd}$ -lab-bck -dors +coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +front(x) \lor coronal(x) -frnt -cor $+ dorsal(x^1) \stackrel{\text{\tiny def}}{=} + back(x) \lor coronal(x)$ Pl $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{round}(x) \lor \text{labial}(x)$ d $-coronal(x^1) \stackrel{\text{\tiny def}}{=} -front(x) \lor coronal(x)$ $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{back}(x) \lor \operatorname{coronal}(x)$ +lab+lab-cor-cor-dors -dorsC-place(x^1) $\stackrel{\text{\tiny def}}{=}$ Place(x) V-place(x^2) $\stackrel{\text{\tiny def}}{=}$ rt(x) $parent(x^{1}) \stackrel{\text{\tiny def}}{=} \begin{cases} \left(parent(x) \right)^{1} \Leftrightarrow \neg \text{vowelFeature}(x) \\ \left(parent(x) \right)^{2} \Leftrightarrow \text{vowelFeature}(x) \end{cases}$ $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

☆ ☆ the transduction: v-features \rightarrow unified v-features unified /p/ C2C1 $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +round(x) \lor labial(x) -lab -rnd -bck -dors +coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +front(x) \lor coronal(x) -cor-frnt $+ \operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} + \operatorname{back}(x) \lor \operatorname{coronal}(x)$ C-pl Pl $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{round}(x) \lor \text{labial}(x)$ d $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{front}(x) \lor \operatorname{coronal}(x)$ +lab+lab-cor $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{back}(x) \lor \operatorname{coronal}(x)$ -cor -dors -dorsC-place(x^1) $\stackrel{\text{\tiny def}}{=}$ Place(x) V-place(x^2) $\stackrel{\text{\tiny def}}{=}$ rt(x) $parent(x^{1}) \stackrel{\text{\tiny def}}{=} \begin{cases} \left(parent(x) \right)^{1} \Leftrightarrow \neg \text{vowelFeature}(x) \\ \left(parent(x) \right)^{2} \Leftrightarrow \text{vowelFeature}(x) \end{cases}$ $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

the transduction: v-features \rightarrow unified



☆ ☆

the transduction: v-features \rightarrow unified v-features unified /p/ C2C1V-pl \mathbf{rt} $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +round(x) \lor labial(x) -lab -rnd -bck -dors+coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +front(x) \lor coronal(x) -cor-frnt $+ dorsal(x^1) \stackrel{\text{\tiny def}}{=} + back(x) \lor coronal(x)$ C-pl \mathbf{Pl} $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{round}(x) \lor \text{labial}(x)$ $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{front}(x) \lor \operatorname{coronal}(x)$ р +lab+lab-cor $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{back}(x) \lor \operatorname{coronal}(x)$ -cor-dors -dorsC-place(x^1) $\stackrel{\text{\tiny def}}{=}$ Place(x) V-place(x^2) $\stackrel{\text{\tiny def}}{=}$ rt(x) $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x) \right)^{1} \Leftrightarrow \neg \text{vowelFeature}(x) \\ \left(parent(x) \right)^{2} \Leftrightarrow \text{vowelFeature}(x) \end{cases}$ $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

vowelFeature(x) = +round(x) V -round(x) V +front(x) V -front(x) V +back(x) V -back(x)

the transduction: v-features \rightarrow unified v-features unified /p/ C2C1V-pl \mathbf{rt} $\operatorname{rt}(x^1) \stackrel{\text{\tiny def}}{=} \operatorname{rt}(x)$ +labial(x^1) $\stackrel{\text{\tiny def}}{=}$ +round(x) \lor labial(x) -lab $-\mathrm{rnd}$ -bck +coronal(x^1) $\stackrel{\text{\tiny def}}{=}$ +front(x) \lor coronal(x) -dors-frnt -cor $+ dorsal(x^1) \stackrel{\text{\tiny def}}{=} + back(x) \lor coronal(x)$ C-pl \mathbf{Pl} $-\text{labial}(x^1) \stackrel{\text{\tiny def}}{=} -\text{round}(x) \lor \text{labial}(x)$ $-\operatorname{coronal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{front}(x) \lor \operatorname{coronal}(x)$ р 42 +lab22 +lab-cor $-\operatorname{dorsal}(x^1) \stackrel{\text{\tiny def}}{=} -\operatorname{back}(x) \lor \operatorname{coronal}(x)$ -cor-dors -dorsC-place(x^1) $\stackrel{\text{\tiny def}}{=}$ Place(x) V-place(x^2) $\stackrel{\text{\tiny def}}{=}$ rt(x) $parent(x^{1}) \stackrel{\text{def}}{=} \begin{cases} \left(parent(x) \right)^{1} \Leftrightarrow \neg \text{vowelFeature}(x) \\ \left(parent(x) \right)^{2} \Leftrightarrow \text{vowelFeature}(x) \end{cases}$ $parent(x^2) \stackrel{\text{\tiny def}}{=} \{x^1 \Leftrightarrow \operatorname{rt}(x)\}$

vowelFeature(x) = +round(x) V -round(x) V +front(x) V -front(x) V +back(x) V -back(x)





unified and v-features are QF-bi-interpretable

and are therefore notational variants?

"The paper capitalises on structural similarities apparent in the Yip and Bao models to show that one can be freely translated into another, and *vice versa*. Such a translation does not result in any loss of the contrasts expressible by either theory. Given these two results, the main claim of the paper is that the two representational proposals do not constitute distinct theories, but are notationally equivalent."

QF is a weak logical language limited to local operations. QF-bi-interpretability can therefore be considered an indication of notational equivalence." (Strother-Garcia 2019: 39)

"A QF transduction is extremely restricted in the degree

to which the output can differ from the input because

(Oakden 2021: 258)

enumerating natural class extensions

general procedure: (e.g. how I currently have it programmed in a python script)

initialize **natural_classes** as dict

- for each licit structure **s** in some theory **T**:
 - for each factor (connected substructure) \mathbf{f} of \mathbf{s} :
 - add f to natural_classes if f not in natural_classes
- for each factor **f** in **natural_classes**:
 - for each segment s
 - if f is a factor of **s**:
 - add s to natural_classes[f]

collect factors

collect segments with each factor



enumerating natural class extensions



full range of contrasts considered:
 {p, t, k, pw, tw, kw, pj, tj, kj, py, ty, ky, kp, tp, kt, y, u, i, i, u}

plain consonants palatalized consonants double articulations unrounded vowels labialized consonants velarized consonants rounded vowels

- by design, unified and v-features do not predict the same natural classes
- but we'll look anyway

natural classes unique to unified



- [-dors]:
 - ('i', 'k', 'kj', 'kp', 'kt', 'kw', 'p', 'pj', 'pw', 'py', 't', 'tj', 'tp', 'tw', 'ty', 'u', 'y', 'i', 'w', 'u'
- [+dors]: – ('k', 'kj', 'kp', 'kt', 'kw', 'ky', 'py', 'ty', 'u', 'w')
- [+cor]:
 ('i', 'kj', 'kt', 'pj', 't', 'tj', 'tp', 'tw', 'ty', 'y')
- [+lab]: - ('kp', 'kw', 'p', 'pj', 'pw', 'py', 'tp', 'tw', 'u', 'y', 'ʉ')
- [-lab]:
 ('i', 'k', 'kj', 'kp', 'kt', 'kw', 'ky', 'p', 'pj', 'py', 't', 'tj', 'tp', 'tw', 'ty', 'u', 'y', 'i', 'w', 'u')

comparing natural class extensions





comparing natural class extensions

 ${p, p^w, t}$

{p, u}

 $\{u, t\}$

 ${p^w, t}$

 ${p, u, t}$



 $NCE_{unified} \neq NCE_{v-feature}$ ${}^{\{p, u, p^w, t\}}_{\{p, p^w, t\}}$ ${}^{\{p, u, p^w\}} transdu{}^{\{tj, w\}}_{\{u, p^w\}}n between them$ is not natural class preserving

|--|





comparing theories



(1) Conditions for notational equivalence

- a. Two models do not differ in their empirical predictions.
- b. Two models represent the same set of abstract properties, differing only superficially.

(from Oakden 2021, summarizing Fromkin 2010)

 if we take seriously assumptions like sharing is caring \$\$,
 then a QF-bi-interpretable contrast-preserving transduction is not enough to satisfy (1a) above introduction foundations example transduction empirical basis existing transductions conclusion

(some) existing transductions



transduction	logic	contrast preserving	natural class preserving
unified vs. v-features	QF	yes	no
Oakden (2021)	QF	yes	no
Danis & Jardine (2019)	FO	yes*	??
Cahill & Parkinson (1997)**	QF	yes	yes

* the segments in the transduction are those that were optima in Shih & Inkelas (2019), but the general set of contrasts are likely distinct

** this was not given as a transduction proper, but it is simple to construct one from their claim

Oakden (2021) & tonal geometry



Yip Bao Yip σ σ σ U h/lh/lС +h/l

from Oakden (2021: 263)

Bao

 σ

h/l

Oakden (2021) & tonal geometry

tone

high

rising

MH

low

ML

low

LM













- Oakden (2021) provides a nonsize-preserving QF transduction (above) between two theories of tone sandhi (left), arguing for notational equivalence
- is this transduction also natural class preserving?

Oakden (2021) & tonal geometry

- Oakden (2021) provides a nonsize-preserving QF transduction (above) between two theories of tone sandhi (left), arguing for notational equivalence
- is this transduction also natural class preserving?





Oakden (2021)

- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's





Oakden (2021)

- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's





Oakden (2021)

- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's





Cahill & Parkinson (1997) & geometric relations

- autosegmental phonology/feature geometry:
 - segments are trees which organize features into constituents
 - constituents predict spreading behavior

- Feature Class Theory (Padgett 1995a; Padgett 2002; Padgett 1995b):
 - segments have trivial structure
 - features are contained in nested sets
 - violable constraints predict class behavior

Cahill & Parkinson (1997) & geometric relations

(3) The *Place* class as a set of sets.

The transition from (3) to (4) is one of notation only.



LABEL(x) $\stackrel{\text{def}}{=}$ LABEL(x) parent(x) $\stackrel{\text{def}}{=}$ included-in(x)





Cahill & Parkinson (1997) & geometric relations

(3) The Place class as a set of sets.



Phonology needs geometry: Implicit axioms in segmental representation

Nick Danis nsdanis@wustl.edu

Main Points -

- Phonological features are organized into "motivated subsets".
- Can a specific feature be in multiple subsets (or classes), depending on the segment, or is all membership unique and absolute?
- The question here is not of implementation (e.g. sets vs. trees), but rather on the implicit axioms governing the definitions of the sets: is class membership globally assigned or locally (per segment)?
- (One aspect of) of Feature Geometry is the idea that segments have nontrivial structure.
- Evidence from cross-cateogry place interactions supports a segment-specific (geometric) model of segmental representation.

Definitions -----

Naturalness of Assimilation (NoA)

Output of assimilation includes two segments having the same feature (value):

$$X \to \alpha F / \left\{ \begin{array}{c} -\alpha F \\ \alpha F \end{array} \right\} \qquad \qquad Agree[F]$$

Geometry There exists organizational information about features that must be specified on a per-segment basis

Global Class Assignment (GCA)

 $(\forall f,g) \left[\texttt{label}(f) = \texttt{label}(g) \to (\neg \exists C) [C(f) \land \neg C(g)] \right]$

"If two features f and g are the same (share a label), their class memberships are always identical."

Unpacking the GCA ------

- Feature organization is hierarchical (Clements 1985, Sagey 1986, a.o.)
- Classes refers to defined subsets of features, *agnostic* of dominating nodes vs. sets

Place = {lab, cor, dors, ...}



- lab cor dors ...
- The GCA is an axiom (potentially) governing how the classes are defined, not how they are implemented structurally
- Given an indivual feature, is all class membership determined irrespective of any individual segment?
- Feature theories can be grouped into those that obey the GCA and those that do not

Case Study: [labial] —

To what extent are these groups of segments related phonologically?
 Rounded vocalics Plain labials y u u k^w / *p* kp /

Feature Class Theory: Obeys GCA

- "Disembodied" feature organization (Padgett 1995, 2002)
 - Rounded vocalics = [+round]
 - Plain labials = [labial]
 - Elsewhere in theory:
 - $[+round] \in V-Place$ $[labial] \in (C-)Place$
- Structure can be removed from individual segments as long as class definitions obey GCA
- Not all theories of FG can be translated into FCT (contra Cahill and Parkinson 1997)
- · Rounded vocalics and plain labials not a natural class
- Other GCA-obeying theories (non-exhaustive): Chomsky and Halle (1968) (trivially), Ní Chiosáin and Padgett (1993), Halle et al. (2000)

Unified Feature Theory: Incompatible with GCA

• Unified Feature Theory: Rounded vocalics and plain labials form a natural class (Clements and Hume 1995)



- The class membership of [labial] can vary segment to segment!
- Unified Feature Theory are incompatible with the GCA (and therefore with Feature Class Theory)
- Other GCA-breaking theories (non-exhaustive): Mester 1986, Padgett 1994, Dependency Phonology, Governmet Phonology

Summary

- In order for rounded vocalics and plain labials to be a natural class, we must assume Unified Feature Theory
- Unified Feature Theory is incompatible with the GCA
- Is there phonological evidence for a natural class of plain labials and rounded vocalics?

St.Louis Washington University in St.Louis

Natural classhood of labials -

• Vietnamese: $k \rightarrow \widehat{kp} / o,u$ _____ (Kirby 2011 a.o.)

$\boxed{V{\downarrow}C{\rightarrow}}$	Palatal	Velar	Labial-Velar
Front	[sec] 'slanting'	*[ek]	*[ekp]
Central	*[ac]	[sak] 'corpse'	*[akp]
Back	*[oc]	*[ok]	[sokp] 'shock'

- UFT: Trigger and target of assimilation are both [labial]
- [labial] V-place triggers [labial] C-place
- Assimilation is natural
- FCT: Trigger is [+round], target is [labial]
- [+round] triggers [labial]
- Assimilation **not** natural
- Related processes:

– Mumuye: $[\widehat{\mathrm{kp}}] \sim [\mathrm{kw}]$	(Shimizu 1983
- Aghem: $b \rightarrow \widehat{gb} / o$	(Hyman 1979

- In order to preserve Naturalness of Assimilation, rounded vocalics and plain labials must be a natural class.
- Natural classhood of labials is only possible assuming UFT.
- If we assume UFT, then the GCA cannot be maintained.
- Thus, organizational structure of these place features must be specified on a segment-specific basis.
- Thus, phonology needs geometry.

References and Acknowledgements -----

Thanks to the Wash U Linguistics Brown Bag audience, many others unnamed here. Chill, Michael and Frederick Parkinson (1997). "Partial Class Behavior and Feature Geometry: Remarks on Feature Class Theory". In: Kyomi Kusumoo, ed. Proceedings of the North East Linguistic Society 27: Annexts, MA: CLASA, pp. 79–91. Chomsky, Noam and Morris Hall (1986). The Source's In: Phonology 21, pp. 225–225. Classical Society 22, pp. 225–225. Classical Society 22, pp. 225–225. Classical Society 21, pp. 225–225. Classical Society 21, pp. 225–225. Classical Society 21, pp. 225–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 225–225. Classical Society 21, pp. 237–444. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 225–245. Classical Society 21, pp. 237–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 225–245. Classical Society 21, pp. 237–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 225–249. Classical Society 21, pp. 237–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 237–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 237–244. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 237–240. Hymm, ed. 2600. "On Fourier Using Society 21, pp. 237–240. Hymm, 2400. Hymm, 2400.

Danis, Nick. 2020. Phonology needs geometry: Implicit axioms in segmental representation. 2020 Annual Meeting of the LSA. Poster.



introduction definitions example transduction existing transductions **broader significance** conclusion

strong generative capacity



" The study of strong generative capacity is related to the study of descriptive adequacy, in the sense defined. A grammar is descriptively adequate if it strongly generates the correct set of structural descriptions. A theory is descriptively adequate if its strong generative capacity includes the system of structural descriptions for each natural language; otherwise, it is descriptively inadequate."

(Chomsky 1969: 60)

strong generative capacity



- ...in syntax:
 - Chomsky's definition often criticized (see Miller 1999 and references therein)
 - Miller (1999) reworks definition of SGC for syntax in robust model theory
- ...in morphology:
 - Dolatian et al. (2021) define and show divergence of WGC and SGC for various morphological processes and their transductions
- ...in phonology:
 - "In morphology and phonology, there are fewer debates on generative capacity. We speculate that this is due to two issues. First, morphology and phonology have comparatively restrictive WGC. Second, it is unclear what external basis (grounding) should be used for SGC, and thus what diagnostics or metrics to use." (Dolatian et al. 2021: 229)

strong generative capacity



- **natural class preservation** should be in the set of diagnostics for evaluating the SGC of phonological theories
- **contrast preservation** is a weaker notion, entailed by natural class preservation
 - Proof: assume two theories are natural class preserving. if they are natural class preserving, they have the same extensions of atomic segments (by def.). if these elements are flattened to a single set for both theories, then S1 = S2. this is the definition of contrast preserving. therefore the two theories are contrast preserving.
- contrast preservation might be an indicator of the WGC

introduction definitions example transduction existing transductions broader significance conclusion

summary



- assumptions about subsegmental structure predicts sets of segments that share structure (natural classes)
- assumptions about computation include the desire for processes to target natural classes, and for certain processes like assimilation to have further restrictions on natural classes (sharing is caring \$\$)
- logical equivalence between **representations** might ignore these assumptions about **computation**
- natural class preservation serves as a proxy for how computation behaves with respect to representation, and is a criterion for a stronger notion of logical & *linguistic* equivalence

going forward



- the definition of *natural class preserving* is based on the representation themselves—can this property be identified by investigating the transduction rules alone?
 - disjunctive labeling
 - loss of labels
- how *else* can transductions themselves be compared and evaluated from a linguistic standpoint?
- how strongly should our metatheoretical assumptions and expectations about linguistic processes be formalized? (or even said aloud)



thank you $\mathfrak{K} \leftrightarrow \mathfrak{S}$

and thank you Adam Jardine (for first helping with the transductions like 4 years ago) & the audiences at the Stony Brook Workshop on Model Theory in Phonology on Sept. 25 & the Wash U Linguistics Brown Bag on Sept. 16