Logical transductions are not sufficient for notational equivalence

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what makes two phonological representations equivalent?

- one answer from mathematical linguistics: two representations are *notational variants* if there exists a bi-interpretable quantifier-free logical transduction between them (Strother-Garcia 2019; Oakden 2020; Danis and Jardine 2019)
- essentially, if a list of rules under a restricted form of knowledge can define all the structure of one model based on the structure of another (and vice versa), then the models are equivalent
- **however**, differences that linguistic grammars care about, such as predicted sets of natural classes, can survive the transduction, therefore QF bi-interpretiblity alone, while important, is not sufficient for a linguistically relevant notational equivalence

a strong generative capacity for phonology

- under the framework of Miller (2001), strong equivalence is relativized to particular domains such that, for a given interpretation domain (ID), the output of an interpretation function IF for some model m_1 ($IF_{m_1 \to ID}$) maps to the same object as the IF for another model m_2 in that same domain ($IF_{m_1 \to ID} = IF_{m_2 \to ID}$). two potential domains are given below:

contrast preservation both models capture the same set of basic contrasts

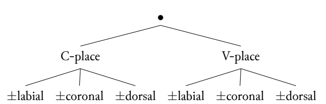
$$IF_{uni \to C} = IF_{v\text{-}feat \to C} = \{p,t,k,u,i,a,\dots\}$$

natural class preservation the two models do **not** predict the same set of natural classes

$$IF_{uni \to NC} \supset IF_{v\text{-}feat \to NC}$$

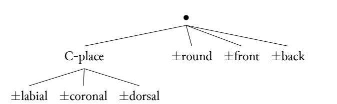
these two models are logically equivalent

unified



same features define vowel and consonant place; dominating node determines phonetic realization (à la Clements and Hume 1995)

v-features



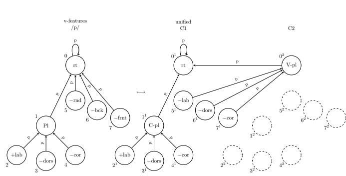
features defining vowel place disjoint from features defining consonant place

- the following is a quantifier-free transduction that translates between the two models
- the models are therefore logically equivalent

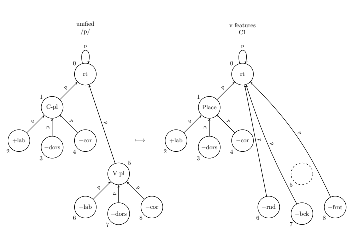
v-features → unified

$$\texttt{rt}(x^1) := \texttt{rt}(x) \quad (1)$$
$$+\texttt{lab}(x^1) := +\texttt{round}(x) \lor +\texttt{lab}(x) \quad (2)$$
$$+\texttt{cor}(x^1) := +\texttt{front}(x) \lor +\texttt{cor}(x) \quad (3)$$
$$+\texttt{dors}(x^1) := +\texttt{back}(x) \lor +\texttt{dors}(x) \quad (4)$$
$$-\texttt{lab}(x^1) := -\texttt{round}(x) \lor -\texttt{lab}(x) \quad (5)$$
$$-\texttt{cor}(x^1) := -\texttt{front}(x) \lor -\texttt{cor}(x) \quad (6)$$
$$-\texttt{dors}(x^1) := -\texttt{back}(x) \lor -\texttt{dors}(x) \quad (7)$$
$$\texttt{C-place}(x^1) := \texttt{Place}(x) \quad (8)$$
$$\texttt{V-place}(x^2) := \texttt{rt}(x) \quad (9)$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^1 \Leftrightarrow \neg\text{vowelFeature}(x) \quad (10)$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \quad (11)$$
$$\textit{parent}(x^2) := x^1 \Leftrightarrow \texttt{rt}(x) \quad (12)$$

$$\text{vowelFeature}(x) = +\texttt{round}(x) \lor +\texttt{front}(x) \lor +\texttt{back}(x) \lor -\texttt{round}(x) \lor -\texttt{front}(x) \lor -\texttt{back}(x)$$



unified → v-features



$$\texttt{rt}(x^1) := \texttt{rt}(x) \quad (13)$$
$$\texttt{Place}(x^1) := \texttt{C-place}(x) \quad (14)$$
$$+\texttt{lab}(x^1) := +\texttt{lab}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (15)$$
$$+\texttt{cor}(x^1) := +\texttt{cor}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (16)$$
$$+\texttt{dors}(x^1) := +\texttt{dors}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (17)$$
$$-\texttt{lab}(x^1) := -\texttt{lab}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (18)$$
$$-\texttt{cor}(x^1) := -\texttt{cor}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (19)$$
$$-\texttt{dors}(x^1) := -\texttt{dors}(x) \land \texttt{C-place}(\textit{parent}(x)) \quad (20)$$
$$+\texttt{round}(x^1) := +\texttt{lab}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (21)$$
$$+\texttt{front}(x^1) := +\texttt{cor}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (22)$$
$$+\texttt{back}(x^1) := +\texttt{dors}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (23)$$
$$-\texttt{round}(x^1) := -\texttt{lab}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (24)$$
$$-\texttt{front}(x^1) := -\texttt{cor}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (25)$$
$$-\texttt{back}(x^1) := -\texttt{dors}(x) \land \texttt{V-place}(\textit{parent}(x)) \quad (26)$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^1 \Leftrightarrow \neg\texttt{V-place}(\textit{parent}(x)) \quad (27)$$
$$\textit{parent}(x^1) := (\textit{parent}(\textit{parent}(x)))^1 \Leftrightarrow \texttt{V-place}(\textit{parent}(x)) \quad (28)$$

but they are not natural class preserving

- every natural class predicted by the v-features model is predicted by the unified model
- there are natural classes predicted in unified model that are **not** predicted by the v-features model

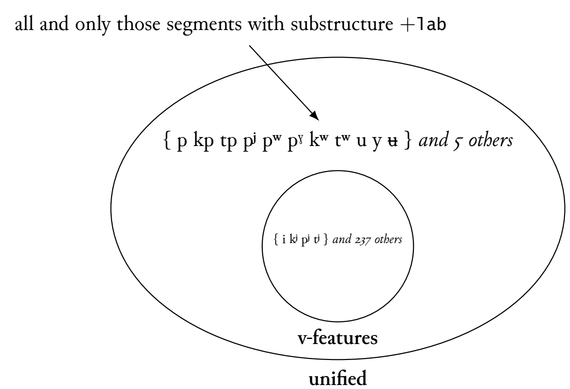


Figure 1: The natural class extensions of the unified and v-features model. The one class shown is defined by the substructure $+\texttt{lab}$; the other 5 are the classes for each value of each place feature labial, coronal, and dorsal.

- this is expected based on the transduction rules of the following form, such as (2):

$$+\texttt{lab}(x^1) := +\texttt{round}(x) \lor +\texttt{lab}(x)$$

- the resulting label on the left side (representing the unified model) is true if either of the two separate labels in v-features are true
- two classes are collapsed into one

full code showing enumeration and comparison of natural classes:
`https://github.com/nickdanis/autosegx`

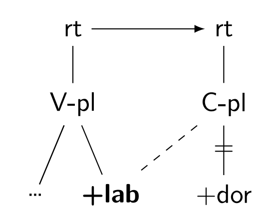
and phonology cares

- case study: /ku/ → [pu]

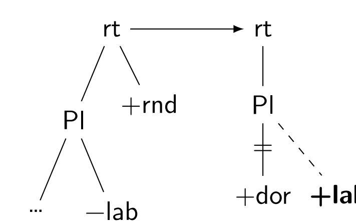
rule-based grammar with spreading

- assume: assimilation is spreading (Goldsmith 1976; Hayes 1986; Clements and Hume 1995)

unified — *v-features*



spreading possible



spreading not possible; requires feature insertion

constraint-based grammar with Agree

- assume: one Agree-style constraint for every natural-class defining substructure in the model (Lombardi 1999; Bakovic 2000)

unified

/uk/	Agr[lab]	F
uk	* W	L
up		*

Agr[+lab] ≫ F

v-features

/uk/	Agr[lab]	Agr[rnd]	F
uk	L	* e	L
up	*	*	*

target candidate is **harmonically bounded**

- the v-features model requires the computational system to utilize a crucially different operation or family of constraints (e.g. *[-lab][+rnd]) in order to capture the same mapping
- regardless of whether these other operations are possible (they most certainly are), the point is **a change in the representational models,** while **keeping grammatical assumptions as consistent as possible,** makes a **tangible** and **nontrivial** change in the predictions

Bakovic, Eric (2000). "Harmony, Dominance and Control". Rutgers University. **Clements, G. N.** and **Elizabeth V. Hume** (1995). "The Internal Organization of Speech Sounds". In: **John Goldsmith, Jason Riggle**, and **Alan C. L. Yu**, eds. *The Handbook of Phonological Theory*. Blackwell Publishing. **Danis, Nick** and **Adam Jardine** (2019). "Q-Theory Representations Are Logically Equivalent to Autosegmental Representations". *Proceedings of the Society for Computation in Linguistics* 2.1, pp. 29–38. **Goldsmith, John A.** (1976). "Autosegmental Phonology". Massachusetts Institute of Technology. **Hayes, Bruce** (1986). "Assimilation as Spreading in Toba Batak". *Linguistic Inquiry* 17.3, pp. 467–499. **Lombardi, Linda** (1999). "Positional Faithfulness and Voicing Assimilation in Optimality Theory". *Natural Language and Linguistic Theory* 17, pp. 267–302. **Miller, Philip H.** (2001). *Strong Generative Capacity: The Semantics of Linguistic Formalism*. Lecture Notes. Center for the Study of Language and Information. 160 pp. **Oakden, Chris** (2020). "Notational Equivalence in Tonal Geometry". *Phonology* 37.2, pp. 257–296. **Strother-Garcia, Kristina** (2019). "Using Model Theory in Phonology: A Novel Characterization of Syllable Structure and Syllabification". Thesis. University of Delaware.